

THE PSYCHOLOGICAL REVIEW

THE EVOLUTION OF BEHAVIOR

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I. BEGINNINGS OF BEHAVIOR

Natural selection is at present the only accredited theory of organic evolution. In accordance with this doctrine what we call life and mind are as much, and as little, accidents as the shape of a stone or the color of an autumn leaf. In our particularly lucky corner of the universe we have happened to occur in accordance with the principle of the origin of organic structures and functions by fortuitous variations and survival of the fittest in the struggle among these variations. That is all science so far has been able to say. It is agreed among scientific biologists and psychologists that the sensitivities, the activities and the attitudes of our ancestors, human and animal, were not merely stages through which they passed, leaving them forever behind, but growths that have somehow entered into the very structure of behavior itself. Far more important than the fossils in the rocks, for a paleontology of behavior, are the tropisms and instincts and emotions we find in ourselves, and the precipitate of the past in an altered environment—in custom and code, institution and tradition, myth and cult, language and literature. And if this inheritance is more noteworthy in so far as it is a human product, it is no less real in the ages which preceded man. Long before our anthropoid ancestors discovered fire and invented the implement and the weapon whereby to mould nature to his ends, animals and plants had been modifying each other and the environment in that process, sometimes competitive, sometimes coöperative, that we call evolution.

Whether the beginnings of behavior are to be carried back to the physical and chemical elements on this earth or whether organisms were imported from another planet, it is perhaps useless, in the present state of our knowledge, to inquire. Arrhenius has suggested that ultra-microscopic bacteria may be driven about the universe by the pressure of light as are the fine particles in the comet's tail, and may have reached our planet by that means. There are known to be species of these microscopic plants which survive any degree of temperature which can be artificially produced, so that the extreme cold of interstellar spaces would be no obstacle. But this, of course, only pushes the problem a step further back. It may be, on the other hand, that within the etheric matrix whence the initial gas-cloud or swarm of meteorites was born from which our earth was derived, the potentialities of life were also generated. Life may have been a spontaneous generation which could take place but once in the primal birth-throes of a planet. However that may be, there evidently was a time in the early history of the earth when only the simplest unicellular organisms could exist. And these were the common ancestors of both the animals and the plants. Allied forms existing today are known as zoöphytes.

In order to understand the evolution of behavior we must employ for comparison some fundamental activity or activities which man shares with the lowest organisms. Such are found in the nutritive and the reproductive functions. Plants and animals, from the lowest to the highest, are characterized by the ingestion, digestion, and assimilation of food, the exchange of oxygen and carbon dioxide with the surrounding medium of the water or the air, and the excretion of noxious matters resulting from the oxidation of the digestive and respiratory functions. They are characterized also by some form of the process of reproduction, either asexual or sexual. It is obvious that the behavior of the lower forms is determined almost entirely, if not wholly, by these two processes. But it is equally true that that complex of behavior patterns which we call the civilization of man rests upon this same twofold basis. The hunting and fighting patterns still

underlie the food and the sex quest. From the food impulse spring the economic, the military, the political, the vocational activities. From the sex impulse, at least in part, spring art, religion, and the home. Literature, education, philosophy, science serve both ends. The food process and the sex process both involve the seeking of the distant object, movement toward it, grappling with it, and appropriating it. The rational processes of the higher animals and of man are but refinements in details of these fundamental coördinations of pursuit, combat and mastery of means to ends.

2. PLANT AND ANIMAL

In their simplest forms plants and animals are indistinguishable, so that many of the single-celled forms are classified in both groups. A plant may be regarded as a sessile animal or an animal may be regarded as a plant which has developed the power of locomotion. The plant has motion but not locomotion. The Myxomycetes or Mycetozoa—Slime-moulds—by some regarded as plants and by others as animals, illustrate an intermediate state, since in their earlier stages they have the characters of free-moving Protozoans, while in their later development they resemble Fungi. Here we see a degeneracy, so far as motility is concerned, due to the assumption of a sessile life. Cope argued from this that all plants are degenerate descendants of Protozoan animals. Probably a truer view traces them to a common source. Animals are not to be regarded as having developed in a linear order from plants. The higher types of behavior as found in the animals evidently sprang, not from the sessile multicellular plant, but rather from the primitive free-moving unicellular animal-plants. The complex sessile plants and the complex animals capable of locomotion evolved in different directions from a common free-moving one-celled ancestor. Nor must we forget, that, while the plants do not develop locomotion, yet their growth processes involve movements. As a rule in the plant these movements are confined to a rhythmic gyration of its growing-points. The tips of the roots and the terminal twigs of the branches of the tree are

continually in motion in what has been called a spiral squirm. The rootlet is searching for moisture in accordance with its hydrotropic proclivities, while the tendrils and buds and leaves are positively phototropic. If these motions could be projected upon a great chart in the sky there would appear an infinitely intricate intertwined pattern of spiral curves, an expression of the fact that even the life of the plant is a never-ceasing movement, from the swelling germ in the seed to the final flowering, fruition and decay of the mature individual.

The plant possesses one function which is absent in most animals: that of deriving its sustenance directly from inorganic nature, especially that of utilizing energy directly from the sun's rays for the breaking up of the dioxides, CO_2 and H_2O , and turning them into vegetable compounds such as starch. The life-process of the plant begins with the dioxides—the carbon dioxide, which is breathed in by the leaves from the air, and the water, laden with mineral salts, which is sucked up from the soil by the roots—disintegrates them, builds up their elements into starches and sugars and other carbohydrates, and into hydrocarbons such as the vegetable oils, and then in the growth processes of the plant breaks them down into carbon dioxide and water again. The same is true of all animal life, except that there is relatively greater expenditure, and less storing up of energy in the case of the animal. But for both the life-process is a cycle from dioxide to dioxide.

This is a wider statement than that which we commonly associate with Behavior. Such processes are usually explained by chemical formulæ. There is no reason, so far as method is concerned, why we should stop with the chlorophyll-bearing zoöphytes in our description of behavior; indeed, the molecule and the atom may some day come within the purview of a science of behavior, especially in the light of the recent discoveries of the complexity of the mechanisms within the cell and the play of electrons within the atom. But in view of the present limitations of our knowledge we begin with the aspect of the metabolic process which, in the animal,

is indubitably associated with an evolution of progressively complicated behavior patterns. This aspect is that of the expenditure of energy which, while common to both plants and animals, is proportionately greater in the latter. Of course all living organisms expend energy. Even the green plants must expend energy in resisting the wind, overcoming the pull of gravity, carrying the sap from the roots to the leaves and conversely, and in the building up of the hydrocarbons and carbohydrates in their tissues. And in so far as they expend energy in these ways, like the animals they breathe out carbon dioxide. But the energy which the green plants expend and the carbon dioxide which they expire is slight in comparison with the energy they store up and the amount of oxygen they dissociate from the carbon of carbon-dioxide. Like a grain-elevator, a little energy is expended to store up a great deal. Compared with the plant the animal, on the other hand, stores up energy to only a slight extent, depending mainly on the food taken in from day to day for the sources of its energy. There is a certain amount of storage in the liver, the muscles, and in adipose tissue, but the distinguishing characteristic of the animal is expenditure of energy rather than its storage.

Even in the case of the animal, not all phases of expenditure are directly correlated with an advancing behavior mechanism, but chiefly those concerned with the sensori-motor adjustments and locomotion—the setting free in the muscles of energy which is available for tentative and overt movements of all sorts. In a broad conception of the matter, the behavior of the animal implies not only the process of photosynthesis in the plant but the electromagnetic vibrations from the sun which we call light and many other conditions in the environment. But in a narrower view, which endeavors to isolate the psychological problem from those of the neighboring sciences, the evolution of behavior may be considered as beginning in the katabolism of the animal organism.

3. EARLY LIFE ON THE SEAFLOOR

The primitive organisms were undoubtedly pelagic forms. It must be remembered that in early geological times there

was relatively less land and more water, that there were no great mountains and therefore less depth of ocean. Today unicellular water-forms like diatoms exist under the simplest conditions of life that can well be conceived. On the surface of tropical seas and lakes and ponds where there is an even distribution of temperature and illumination such plant life exists in enormous quantities. Under such conditions there is no necessity for the development of organs which will enable the creature to perceive the distant object. All it needs is immediately about it so evenly distributed that no special organs of perception or manipulation are required. It is hard to see how the more complex types would ever have arisen if the environment had continued to be thus simple. Brooks thinks that the appearance of a habitable bottom of the ocean was the occasion for the first development of the higher forms. The first stretches of water to be habitable were probably surface areas. But upon the seafloor there would gradually accumulate a richer medium than was to be found upon the surface, as the detritus of organic waste, food-particles, etc., would be deposited there. Animal forms dependent upon this food deposit would not have to be free-moving like the forms on the surface, nor would they need to be chlorophyl-bearing. They would become sessile and build up into colonies, at first simply aggregates of unicellular forms cohering, as they subdivided, in more or less solid groups like the sponge and the coral. Then those at the centre of such groups would change in structure and we would have the first appearance of two layers of cells, an outer sensitive layer and an inner absorptive layer, which would take on the perceptive and assimilative functions respectively. Here, in this symbiosis of single-celled Protozoans we get the first appearance of the metazoa or many-celled animals.

The sea is truly the mother of all life. All cells, even those of the many-celled animals and plants, require a liquid medium in which to live. The cells which make up the structure of an ox or a man are as truly water-forms as the diatom or the amœba, since they are all embathed in lymph.

This is as true of bone-cells and muscle-cells and nerve-cells as it is of the red and white cells which swim in the blood. In the case of the many-celled animal the liquid medium has been folded into the interior of the body. This has been rudely schematized by saying that if we were to take all the cells of the body and spread them out over the surface of the ocean they would have to spread over an extent which would include as much nourishment as is represented in the liquids of the body. When the water-form became a land-form it carried its liquid environment with it in the form of the lymph of the blood. The most important distinction between the unicellular and the multicellular form is, therefore, that the multicellular form controls the medium which surrounds its cells, while the unicellular form merely moves through a medium over which it has no control, picking up such food as chance throws in its way. The multicellular form keeps its internal liquid medium at a certain temperature, restores it regularly by breathing in oxygen and by the digestion of food, while the unicellular form is at the mercy of the elements.

The most primitive forms which represent the transition from the colony to the pluricellular form are those which are simply a sphere of cells surrounding an inner cavity, like a rubber ball. This, in the development of the mammalian embryo, is called the blastula stage, where we apparently have a hint of what took place in the evolution of the race. The next stage corresponds to the poking in of one side of this rubber ball, thus producing the gastrula or stomach-like stage. This second cavity, resulting from the invagination of the blastula stage, is thus really a part of the outside world folded in to become an alimentary tract. By this method not only is the outer liquid medium transferred to what are now interior cavities of the organism, but other structures and functions originally on the exterior of the body are carried within to constitute the deep-lying sense-organs and nerves known as intero- and proprioceptors.

4. BILATERAL SYMMETRY AND RESPONSE TO THE DISTANT OBJECT

With the development of the multicellular form we first encounter the differentiation of definite organs for the perception and appropriation of food and mates. Locomotion, likewise, emerges at the point where, in the evolution of the sessile animal, its demands exceed the capacity of the immediate environment to supply nourishment. It is the problem presented by this combination of conditions that determines the appearance of sensorimotor structures and functions answering to what has been called the recession of the stimulus. As the animal form becomes adapted to an increasingly wider and remoter environment, the receding stimulus is represented in behavior by the conditioning of the reflex: for every complication of the context the action-systems of the organism exhibit a corresponding cross-reference synaptic or protoplasmic. Thus we may conceive to have arisen the twin facts of bilateral symmetry and response to the distant object.

The route by which the free-moving multicellular animal form evolved from its free-moving unicellular ancestor is far from clear, but it may be conjectured that in part this took place by a detour through the sessile colony types or compound individuals whose original habitat was the seafloor. With the relative impoverishment of the liquid medium in which such forms subsist, the occasion would arise for the detachment of such forms and the resumption of some mode of locomotion which, teleologically interpreted, might be described as the pursuit of the wonted stimulus. The free-moving multicellular form returns once more to the surface in the search for food, solid and gaseous as well as liquid. The jelly-fish would seem to represent such a transitional type, since its first stages are sessile while its later stages are spent on or near the surface as a free-moving form.

Without presupposing any entelechy, or any resident principle other than such as are suggested by the facts of geotaxis and chemotropism, we may yet clearly conceive the possibility of elongation of the irregular shaped multicellular

mass into a concatenated and later into a metameric form, with clearly defined head and tail ends—due, on purely mechanical principles, to the influence of localized stimulations in the environment. The results of experiments with viscous substances externally propelled through a liquid or semi-liquid medium go far to support the conclusion that more rapid movement and continual adjustment of a plastic protoplasmic mechanism, in its progressive adaptations to the chance canalizations of stimulus in a fortuitous environment, would tend to produce the various types of bilateral symmetry which as a fact we do find.

Whether or not the existing radiates, such as the sea-urchin and star-fish, represent an arrested stage in this transformation, we are warranted in supposing that such spherical or radiate structures represent one of the abortive, while yet partially successful, solutions of this problem. The jelly-fish moves about slowly by simple rhythmic pulsations of its umbrella, without any structural characteristic to determine that it shall move in one direction rather than another. It is still relatively at the mercy of its environment. The more rapidly moving forms, on the other hand, take on an elongated shape, bilaterally symmetrical, and later, as definite organs for perceiving and manipulating the distant object develop, we have the appearance of the head and tail ends of the animal. At first any part of an organism will serve as a mouth, as in the case of the *amœba*. But with the development of the elongated form, such as the worm, the mouth is found in that part of the body which first comes in contact with the food-object. It would thus be the character of the objects toward which the animal is moving, and indeed the character of the environment in general, which would determine the appearance of the bilateral form.

The mode of origin of the segmented or metameric form is still shrouded in mystery, and here we encounter one of the most baffling and enticing of the gaps in our account. Without doubt here, as elsewhere, whole chapters remain to be written. But, on the other hand, nowhere are structure and function more obviously related: the exfoliation of receptors

irritable by the receding stimulus is accompanied point for point by an involvement of effectors for pursuit and capture of the end-object. Moreover, in each segment is found a double nerve-knot for transmitting and accelerating the response to the stimulus, while at the head end are the cerebral ganglia for controlling the responses of the segments in the interest of the organism as a whole. Thus a wave of motion passes from one end of the organism to the other, each segment acting more or less independently of its neighbors, and yet capable, when necessary, of coöperating in the more important functions segregated at the anterior end.

5. INVERTEBRATE AND VERTEBRATE

Evolution is not continuous from plant to animal; invertebrate to vertebrate; fish, reptile, bird, mammal to man. There is a concomitant evolution of different types. Evolution is like a tree with a branching trunk, not like a tree with one central stem running straight from tap-root to terminal bud. There are two types of development, as represented by the invertebrate and vertebrate. In each of these we find a relatively complex type of behavior, but based on a distinct principle of structure and function. In the one we find a defensive exoskeleton, a ventral nerve-chain, and remarkably specialized types of invariable response. In the other we find a supporting endoskeleton enclosing a second and new type of nervous system not represented in the former.

In the higher types of invertebrate there is a collapse of the metamerous, the ganglia of various segments being condensed or telescoped into a single double ganglion. The ganglia in the head-end remain much as in the worm, but in other parts of the body, such as the thorax and the abdomen, certain segments disappear or coalesce, and the different nutritive and reproductive processes and locomotion are segregated in these regions. The legs disappear at the head-end where they are transformed into mandibles, jaws, and become concentrated in the thorax, while the assimilative and reproductive systems come to be located in the abdomen. The advantage of this division of labor, as seen, for example,

in the crustaceans and insects, is found in the greater unity and power of direction of activity which such an organism has, contrasted with the unspecialized metameric form in which most of the functions are performed equally by all of the segments. Instead of all the segments moving and digesting and breathing and reproducing, certain ones are specialized for the perception of the distant object, others for movement, others for digestion, breathing, reproduction, etc. Contrast the spider or the bee or the ant with the worm, in this regard. In the bee or the spider the impulse does not have to pass down the whole series of metameres, as in the case of the earthworm, to produce a movement. It is fair to assume that the greater complexity of behavior of such forms is due to this centralization and distribution of functions, the specialization of different regions for different functions and the bringing of all the motor organs into more direct and effective relation with the controlling center in the cerebral ganglia.

In the vertebrate, on the other hand, the metameric form is preserved, as shown in the structure of the central nervous system with its succession of spinal and cranial nerves springing in pairs from the bilaterally symmetrical spinal cord and brain-stem. Here the great development is in the brain at the head-end, in striking contrast with the development as it takes place among the invertebrates. In the vertebrate the unity and direction of action is secured rather by the multiplication of conduction-pathways and synapses than by the telescoping of the metameres. It is as if nature had experimented with both methods of evolving greater variability of behavior but succeeded only with one, in the vertebrates; securing great complexity and precision of response in the case of the invertebrates but at the price of variability. They thus represent parallel, not serial, developments, each reaching a climax along a different line.

It must be remembered, however, that the vertebrate retains an equivalent of the invertebrate nervous system in the autonomic system which, like the ventral system of the invertebrates, consists of a mere ladder-like double-chain of nerve-knots. And since what we call our instinctive and

emotional life is correlated with this structure, it is not mere fantasy to suppose that our behavior is here linked with that of our distant kin among the articulates.

6. CEPHALIZATION OF THE SENSE-ORGANS AND DEVELOPMENT OF THE BRAIN

Three facts characterize the head end of the organism: the aggregation of the sense-organs which recognize the distant object; the centralization in the brain of the nerve-centers which control the movements of locomotion toward the distant object; and the mouth with its mandibles, teeth, etc., for the manipulation, mastication, and ingestion of the food object when reached.

The function of the sense-organs is the recognition of the food or sex object. One of the most noteworthy facts of the structure of the evolving vertebrate is the crowding together in the head of the projicient receptors. What in the lower animals are present as chemical and mechanical receptors scattered more or less generally over the periphery of the body, are here brought together in a closely associated cluster of irritable end-organs. The retina is a group of glorified warm-spots, the cochlea a group of modified touch-spots, while tongue and the nose contain areas which represent an intensification of the chemical irritability still characteristic of the visceral epithelia.

The extraordinary development of the brain, and especially of the cortex, is the inevitable accompaniment of this cephalization of the distance-receptors, since it is through these that the organism becomes related to the remoter parts of the environment. The relative weights of the brain and body as a whole, for example, jumps from the ratio of 1 to 5,668, in the case of fishes, to the ration of 1 to 186 for mammals. And, as the structure of the cortex shows, the greater part of this advance in relative amount and complexity of structure is represented in the ganglia and conduction-pathways necessary to equate the activities of these highly specialized receptor organs. "Fossils show that while the average size of the mammals has diminished since the middle

Tertiary, the size of their brains has increased more than one hundred per cent." (Brooks). The range of possible accommodation to a variable environment increases, and behavior becomes less dependent upon definite heredity and more upon acquired habits of the individual. Cunning counts for more than size and rational deliberation for more than precision of invariable response.

All this specialization of receptors and multiplication of conductors is more or less immediately related to the process of finally bringing the distant object to the mouth, or at least, as in the case of sex, for the sake of manipulating it in such a way as to bring about, sooner or later, the fulfilment of the inherited impulsions. And not only are the cephalized sensorimotor processes, represented in the distance-receptors, subservient to this end, but also all the processes of locomotion and manipulation. The leg was developed for the sake of the jaw. Everything in the baby's hand goes into its mouth. The leg and the wing and the fin are for flight or for the stalking of the prey. And practically all the minor motor organs were specialized to lure or to warn, from the curious fish that dangles a worm-like appendage to its head before its victim, to the cotton-tail rabbit or deer whose conspicuous upturned flag is an instant signal to the group. Nor, indeed, has the predatory device disappeared even from the food quest of man, since the more sagacious invent legal methods of defrauding their fellows of the full product of their labor.

7. TRANSITION TO THE LAND AND THE CONTROL OF THE ENVIRONMENT BY THE ANIMAL

The passage of certain animals from the water to the land was probably determined by very definite conditions. In the water there is comparatively little development of plant life and what there is is unicellular for the most part. The conditions are so uniform that there seems to be no occasion for the development of the higher forms of vegetable life. The animals either feed upon this unicellular plant life or feed upon other animals which in turn feed upon it. The

unicellular plant forms of the plankton, such as the Plantain that covers thousands of square miles in the Sargasso seas, find themselves surrounded by the essentials of their existence and nothing would be gained by the development of a multicellular form.

But along the margin of lake and pond and stream we see the gradual passage of both plants and animals to the land. The algæ, among the plants, we may suppose, crept from the sea to the shore and evolved into the moss and the fern. First the spore-bearing plants and the conifers and later the plants which blossom and mature their seeds in an ovary, establish the new habitat. And among the plants which propagate by cross-pollination of their pistils, obviously the anemophilous must precede the entomophilous, since pollination by the wind would be possible long before the floral organs of plants were modified to correspond with the habits of insects. The transition must have been due to successive changes in the environment. The continents were rising, the seafloor was settling, and the denser atmosphere of the earlier times was becoming rarified. The giant forests of the Carboniferous era, of which our California Redwoods and Sequoias are relics, sufficiently indicate the enormous quantities of carbon dioxide which in those days must have been present in the air to enable plant life to attain to such proportions.

The amphibian of today is a reminder of the transition on the animal side, as are also the Dipnoids or Lung-fishes—the lung originally developing from the air-bladder of the aquatic form. Among the reptiles we encounter the development of the three- and the four-chambered heart, the passage to a gaseous medium necessitating the separating of the arterial from the venous blood. The ancestors of the spiders and the insects, supposed to have been evolved from a tracheate worm, likewise took to a terrestrial existence. Amphibian, reptile, bird, mammal represent the order of evolution of the vertebrates on the land.

In general when the plant passes to the land it precedes the animal, for the animal must have the plant to feed upon.

The animal can go no further from the plant, in its excursions, than the energy taken from the plant in the form of food will allow. But the conditions of the plant on the land are very different from those in the water. The multicellular form now becomes a necessity in order to provide an internal liquid environment for the cells of which its tissues consist. Consequently, in passing to the land the migrating forms must either carry their liquid environment with them or, as in the case of bacteria, usurp the liquid environment of some other organism. The plant must protect itself against evaporation of its sap by the sun or the liquid environment of its cells will be dried up; hence the development of cellulose tissue for protective purposes. Moreover, on land the two sources of energy, air and water, upon which the plant is dependent, are separated, as they are not in the water. The carbon dioxide and sunlight are in the air while the nitrates and potash and phosphates and other mineral solutions are in the soil. The fibrovascular bundles of cellulose tissue not only protect the living layers of cells from evaporation but interconnect the leaves and roots which are in contact with the air and soil respectively. And this same cellulose tissue, in the form of dead bark and woody fiber, protects the cambium layer of living cells from the depredations of the animal.

The passage of the plant to the land, with its changed conditions, accordingly, presents new conditions for the life of the animal. The multicellular plant has erected a barrier; hence the terrestrial animal must develop in corresponding complexity in order to overcome this obstacle to getting its natural food supply. How difficult this was, merely as a physical and chemical problem, is seen in the difference of structure of the digestive organs of beasts of prey and cattle. Practically the entire energy of the ruminant, as represented in its series of stomachs, is devoted to breaking down this cellulose tissue of the plant wherein its nourishment lies. The ox has a gut which is thirty times its own length while the tiger or lion has a gut which is only eight times its own length. This, in a rough way, is an index of the amount of energy which is required for the digestion of the two kinds of food.

There is no necessity for the plant to develop the bilateral form, methods of locomotion or organs of any complexity for the recognition of the distant object. It finds its sources of food everywhere about it in the soil and the atmosphere. But the animal must develop such organs or perish. The main problem, as the animal passes from the water to the land, is to overcome the obstacle presented in getting at and digesting the cellulose green tissue entrenched behind the outer protective layers of woody fiber. Of course, this relation between the plant and the animal is one not only of conflict and struggle but also of coöperation. The grass grows faster for being cropped. Animals are an important means of distribution of the seeds of plants; and in return, we may say, the plant develops its fleshy fruits and edible seeds for the animal. There is both competition and coöperation in nature. And as we pass up the evolutionary scale we find that plant and animal life are increasingly interdependent. The dependence of the plant on animal life reaches its climax in the reaction of the human animal on the plant world in agriculture—a process the same in principle as that which operates on the lower levels.

8. THE BUILDING UP OF A SPATIAL AND TEMPORAL WORLD

The recognition of the distant object and the possibility of movement toward it is the basis on the part of the evolving organism of the building up of a spatial-temporal world—a world which can be stated in terms of motion and locomotion. This represents the going out of the animal beyond its bodily limits to elements which lie outside of itself. As Lotze pointed out, when you take a stick into your hand you enlarge your contact world by the length of the stick. When you place your eye to the telescope or microscope or your ear to the telephone or get aboard a car or ship or aëroplane you extend the function of eyes and ears and legs. This is an important step in the evolution of the animal form—this attempt of the organism to state the environment in terms of the activity, the functions, the behavior, of the organism itself. We are prone to think of the environment as fixed,

but of course there is an evolution of the environment as truly as of the organism, and in all stages evolution is really a progressive dynamic balance of interaction between two changing systems of activity.

In a certain sense we may speak of the animal as standing at the center of a sphere and of this sphere as increasing in diameter as the capacity of its distance-receptors and controlling effectors increases. The head-end with its sense-organs, of course, is directed toward the distant object. We may conceive the fundamental line of space to be the axial line of vision drawn from the theoretical cyclopean eye to the distant object. This line would represent, however, not the diameter, but the radius of the field of activity, since the animal is able to turn about. Space is a spheroid to us because the body is capable of revolving in every direction; and the eyes within their sockets by their rotating movements, in conjunction with the turning of the atlas upon the axis, further facilitates this. If one's eyes were fixed in their sockets, one's head on one's trunk, and one were unable to turn about at the waist or on one's feet, probably space would be a very different affair. It may vary for the sessile animal, for the quadruped, for the burrowing, the climbing, the swimming, the flying form.

But the existence of this axial line would be of importance to the animal only if it were able to move toward the distant object. There must therefore be an equation between the motor processes of locomotion and the length of the line. As the terrestrial animal arises from the earth and acquires organs of locomotion, either as a quadruped or as a biped, there arises the problem of the maintenance of balance while in motion—a problem which appears in a different form to a walking, a leaping, a swimming, and a flying form. This problem increases in complexity as we ascend the scale from the creeping myriapod with its fabled thousands of feet to the human being which has only two supports. The advantage of the fewer supports and the upright attitude of the biped is the possibility of more rapid movement and a wider sweep of vision. The statement of the environment

in terms of the maintaining of such a balance gives us another fundamental equation of the space-time world of the animal.

The distance between the organism and the distant object is measured in terms of the number of paces or other functions of effectors necessary to reach it, and the animal interprets the one in terms of the other. This is a fundamental equation in the building up of a space and time world. The possibility of analyzing this line in terms of the locomotor processes and conversely, is basic to all space measurements and time adjustments of living creatures. Inasmuch as the animal never has a single object only in the field of vision, this implies in some sense a selection of one object or situation as the objective point and the treatment of other objects in the field of vision either as obstacles to be overcome or as instruments to the end represented by this selection. All the stimulations are necessary to the fulfilment of the act as a whole—even those that are subordinated to the main activity; we define ourselves in terms of what we inhibit and reject as truly as in terms of what we attend to and select. The ground, for example, represents the resistance offered to the foot, but it represents also the means for walking or leaping forward. The trees and bushes may be used as screens for stalking the prey. The intelligence of the animal is in direct ratio to its capacity for transforming these negative obstructions into positive aids to what it is undertaking to do.

9. THE SHIFT IN EVOLUTION WITH THE APPEARANCE OF MAN

The end toward which evolution moves in the animal kingdom seems to be control over the environment, especially over the vegetable world. The animal, like the plant, is confined to the circular chemical process from dioxide to dioxide; but the animals make the plants partially prepare their food for them, and some animals, the carnivores, make other animals still further prepare their food. This process exhibits higher and higher forms of behavior as there is more and more control over the expenditure side of this circle. When in civilized man we get an adequate control over the

vegetable and animal environment the evolutionary movement along this line reaches its climax and a certain degree of finality. Unless some great catastrophe transforms the face of nature it seems as if this stage of evolution, which Darwin and his followers so adequately grasped, would give place to a new type of development.

For one thing, evolution has shifted from the development of sensory and motor organs to a modification of the environment itself. Man does not evolve a better organ of vision but invents lenses to supplement his imperfect optical instruments. He supplements the ear by telephone and telegraph. He refines his tactile delicacy by instruments of precision. He supplements motor and locomotor organs by artificial means of power and transportation. Evolution shifts from the inside to the outside environment. Thus is opened up the era of extra-organic evolution—evolution in terms of implements and weapons and extensions of the sensory and motor functions by mechanical means. It is of course just as legitimate to speak of evolution as taking place in the pushing out of railroads on the frontier, the extension of commerce, the invention of the automobile and the *aéroplane*, as in the cephalization of sense-organs and the telescoping of the metamerous.

The whole life period is estimated to have been anywhere from twenty-five to sixty-five millions of years, almost all of this having elapsed before the appearance of the Primate. Yet this comparatively ill-equipped and puny mammal has transformed the face of the earth, exterminating not only the more ferocious of the beasts, but many varieties of his own species. The polar bear or the cactus tree is relatively adapted to its environment. It must await secular changes to alter its form; and if these are too sudden it will perish. But certain creatures, among whom were man's ancestors, instituted what, looking back upon it from our present vantage-ground, we may call the human as contrasted with the natural economy. The creature controls and modifies the environment instead of being controlled by it. He turns meandering rivers into straight irrigation ditches; he plants

seeds and cultivates them instead of depending on the uncertain sowing by wind and insect; he domesticates animals for food and labor power; he fashions animal pelts and the fibers of plants into clothes and the wood of trees and materials from the quarry and claybank into shelter; he discovers or invents fire, tools and weapons; he is discovering means of immunization against the ravages of harmful parasites; and there only remain the problems of artificial production of protoplasm and interplanetary transit when he will have controlled the fact of death itself.

It was the greater range of control and variability of response which these supplementary extra-organic sensorimotor processes made possible which gave man his great lead over the other types. The insects developed complex forms of sensory and motor organs, sometimes of marvelous delicacy, but this only rendered them still more dependent upon fixed conditions in the environment, whereas the unique thing about the human variation was the use of one part of the environment (the weapon or the tool) to control another part, in the interests of the organism. As one writer says, "It was not to the fact that man possessed hands that he owed his mastery. It was because he used those hands to make an alteration in his environment" (Lane, 'Law of Social Motion,' 120). The higher types may be said to have organized more of the environment into themselves by reason of a more variable adjusting apparatus in the brain whereby an equation might be established among increasingly remote stimulations. Or, again, looking at it from the other point of view, the higher animal may be said to have widened the scope of his individuality or selfhood just to the degree that, by extra-organic means, he has increased his control over the environment by supplementary sensory and motor organs. If by natural selection is meant merely the non-deliberative method of survival in the sub-human stages of evolution, then natural selection is comparatively wasteful, arbitrary, rigid and blind, leading to the survival of the fittest in only a limited sense of that word; but if it is used in the widest sense as including all phases of this extra-organic evolution, then

there is every reason to believe that all the phenomena of human civilization and culture have persisted only because of their superior survival value.

It is conjectured that the comparatively sudden shift in the evolutionary process which we encounter with the appearance of man—sudden in comparison with other geological changes—took place because of the abrupt alteration of the environment when the ice-sheet began to descend from the north in the early part of the Quaternary age. There is evidence that in what are now temperate and arctic zones all animal and plant life has been at some time related to tropical conditions. Coal is found in Iceland and Greenland and in the north of Scotland. The cinnamon tree flourished in Iceland. At the beginning of the ice-age there was a migration of this tropical forest gradually southward. This can be followed in the North American fossils. The animals that were able to migrate with the forest did not change their forms; their descendants are found today in essentially the same forms in South America. The change in form took place in those animals that lagged behind the migrating forest and succeeded in adapting themselves to the changing conditions. There is evidence of this in other animal forms. Any animal which staid behind had either to grow in size, in order to secure the necessary amount of bodily heat, since the surface area increases in diminishing ratio with the volume, or it must develop a heavy coat of fur, or hibernate, or perhaps all of these at once.

But in the Asiatic and European fossils we find a different set of facts. The migration southward of the forest paralleled that in North America. Here too there is relatively little difference between the early forms and those in Africa today, although there is a greater difference than between North and South America. But at the Indian ocean and the Mediterranean sea, or the great inland ocean from which the present Mediterranean is derived, there was a check to the southward migration of the animal forms. In the case of America there was no great barrier to the south to offer obstruction, but in Asia and Europe the conditions were

present for what may be regarded as a crucial experiment in evolution. The North American animals, including the primates, migrated with the forest to the south; the monkey forms in South America today are essentially the same as the fossil forms in the North. But in the peninsulas of Southern Asia and Europe they were penned in, caught in a trap, as it were: they must either undergo a rapid adaptation or perish. The mastodon illustrates a case of adaptation among the lower animals. His great size and heavy coat of hair made it possible for him to remain in the frozen zone. But the primates were relatively small and highly organized and doubtless most of them perished.

If any of them were able to adapt themselves it would have been just where in fact is situated the traditional cradle of the race. On the Indian peninsula there would have been no possibility of further migration to the south, while the Himalayas towered to the north. Here would be the point of struggle; if a new type could develop at all, it would be here. It is conjectured that this is what in fact happened. The form to undergo such a transformation naturally would be one of the primates, since they were the most highly organized and at the same time the most unstable, and therefore the most plastic. We can imagine some one or more species that descended from the trees, gradually changing their diet from fruits and nuts to shell-fish and other forms that could be found on the beach, preying upon other weaker animals, and finally picking up the stone or the club as a weapon, perhaps discovering fire in the chipping of flints, and finally evolving the rude beginnings of clothing and shelter. There is evidence in our own anatomy of a partial change to a carnivorous diet.

Here the beam tips in the evolutionary process. Our ancestor was not equipped to meet such an emergency so far as his gross structure was concerned; many other forms were indeed better equipped than he. What determined his survival and his preponderant power ever since was the accident, if it was an accident, that he began to live by indirect means. The center of the struggle was shifted from

the organism itself to the environment. Extra-organic extensions of the organic functions so transformed the environment in the case of the Primates that they were able to survive where otherwise they would perish. The conquest of the environment by indirect means is the great mutation by which the evolutionary process leaped forward in man.

10. THE HEGEMONY OF THE ACCESSORY MUSCLES

One cannot penetrate into the beginnings of behavior, especially as found in the action-systems which evidently underlie human conduct, without the query arising whether the emergence of that mirror-image of himself that man has come to dignify as a distinct entity under the word mind or spirit or the psychical, may not have been a quite accidental and incidental product of the extraordinary development of the accessory muscles, particularly those of speech. Up to the time when our anthropoid ancestor began to babble to some effect, his motor processes did not differ essentially from those of his brute associates. They involved the gross musculatures of the trunk and limbs and, on the whole, constituted a unified and continuous action-system. But when, in the midst of such fundamental adjustments of food and sex, the freedom of the arms and hands made possible the use of the weapon and the tool, and thus released the voice, measurably, from its strict servitude to the needs of the warning cry and the signal of distress, we may imagine that the mechanism of the larynx came into the service of social ends. With the multiplication of the new types of situation which would spring rapidly from the mastery of the environment through the use of one extra-organic process to control another, the need for some method of communication would arise, more precise than the primitive gamut of insufflations and grunts. What more natural than that this flexible mechanism of vocalization, relatively freed from the sterner demands made upon it in the past, should be modified and enriched to furnish the symbols of reference necessary to a more diversified group activity.

With the recession of the snout, the abbreviation of the

tusks or tearing teeth, and the substitution of the hand for the muzzle in the manipulation of the food object, the finer musculatures situated in the head and neck become liberated for this surrogative function in relation to a widening range of sensorimotor adaptations.

And it is chiefly noteworthy that the use of such an accessory musculature, under the conditions of an extra-organic evolution such as we have been supposing, would naturally represent, in the main, arrested or incipient (*i. e.*, controlled, or what in recent studies have been called conditioned) responses. Speech partakes of the character of an attitude rather than an act. The word or name comes to stand for the act. It is a tentative movement which serves as a kind of substitute for the completed performance. As the distinctively human aspect of behavior, in the course of time, passed over almost entirely into terms of the extra-organic adjustments, these intra-organic coördinations which have come to stand for them, would finally carry an increasing ratio of the meanings of life. And since such meanings are always conditioned, in the sense that they are symbolic of the deeper-lying and more pervasive activities of the fundamental muscles, and ever point to them and presuppose them, it is natural that there should appear a plane of cleavage between the two levels of behavior. This is the biological basis for that duplication of realms that has played such an important, and often disastrous, part in the evolution of psychological theory.

II. THE FUNCTION OF THE LARYNX IN INDIRECT CONTROL

We have seen that the liberation of the larynx was a collateral effect of the eversion of the foot, the freeing of the arms and hands, the recession of the snout, and the consequent substitution—except for the mastication of food—of the manipulative functions of the hand for those of the mouth. This effect was of momentous importance in relation to the initiation of increasingly indirect methods of control. The substitution of the gesture for the completed act, of the spoken word for the gesture, of the written for the spoken

word, and then, within the confines of the now socialized behavior of the individual, of subvocal articulation for interlocutory discourse, introduced a technic for handling the remotest parts of the environment as the stimulus receded farther and farther from the response. No part of overt behavior escaped the effects of this introversion: every act, and every object and situation, representing possibilities of action, was destined to find its counterpart in some incipient innervation or tentative tintinnabulation of the articulo-motor apparatus, and to a less degree of the optico-motor and grapho-motor mechanisms involved in reading and writing.

It was no accident that the elaboration of symbols took place primarily in connection with the functions of the larynx and the ear rather than in connection with those of the eye and the hand. It was these which were first freed from the stress of the struggle for survival. The eye must still be at the service of the hand, but the ear was, comparatively, released from this necessity and could take up with the larynx the functions of speech. Moreover, in our ability to make sounds we have a mechanism which, unlike the signs which appeal to the eye, we have always at our service. In the midst of all other kinds of activity, in any position of the body, in darkness as well as in light, the larynx may keep up its kinæsthetic comment upon the other activities, with a check upon it, in turn, by the ear. This expansion of activities is reflected in the increasing complexity of the central conductors, chiefly in the cortex. It is an instructive fact that the center for language lies adjacent to the center for the hand and without doubt the fact that the language center is unilateral is to be correlated with the fact that man is normally right-handed.

Behavior is turned in upon itself in the form of arrested movements, this inhibitory process promoting, in turn, the multiplication of new conduction-pathways. Speech uses many of the same muscles as eating, but it has elaborated a far more complex system of synapses. The function of this new behavior-complex is what is called thought, which has been significantly described as interior speaking. The advent

of a larynx, converted to the uses of language, gave our ancestor a new machine by which his power of accumulating experience was increased. It is the power of operating at a distance by the use of symbols, which demarks man from the brutes and gives one man or one race superiority over another. Man is *homo sapiens*, the thinking animal, because he is the speaking animal; brutes are the dumb animals.

Brutes are capable of vocal expression, it is true, but the function of symbolization in such expression is minimal. There is a limited amount of expression for the sake of communication—the cry of distress, the call for help or for mates—but for the most part expression is for expression's sake. The animal grunts or whines or barks or roars or chirps or sings with very little precision of reference to what his fellows are doing. The vocalization of man, on the contrary, is characterized by its indicative and social reference. As Romanes said: "So a man means, it matters not by what system of signs he expresses his meaning: the distinction between him and the brutes consists in his ability to mean a proposition." The utility of symbols has been enormously augmented by the arts of writing and printing. Language and literature, libraries, museums, laboratories, all the significant institutions of civilization, are the precipitate of such premeditated acts. The emerging individual finds a vast amount of the work of catching up with the race already done for him. A great part of his learning may be done by proxy. In a certain sense each new generation may begin where the previous one left off. The human infant is much more helpless in its environment than a kitten or a puppy of the same age, but he has what they lack, a budding mechanism of vicarious response by which he soon immeasurably outstrips them.

We do not know precisely how language originated—probably in the cries and calls of animals in relation to food and sex, and the expression of such other fundamental trends as fear and anger and pain. We may observe its beginnings by a study of the child. Such a study gives us an insight into what is perhaps the most complex and closely interwrought

system of behavior-patterns which is anywhere to be found—the only basis of behavior upon which it is conceivable that the vast superstructure of human literature and science and philosophy and art could have been erected. The complexity of this system is apparent in an enumeration of the sensorimotor mechanisms which are involved. A word may be spoken, heard, written or seen, involving the intricate interplay of musculatures of the articulomotor, the auditory, the graphomotor, and the opticomotor apparatus, while, in the synaptic connections in the cortex and lower centers in the brain, are to be found a most complicated system of corresponding conduction-pathways. As in the history of the race, the child begins by hearing words spoken, gradually learning to speak them himself by reason of that extraordinary over-production of movements in vocalization which is the natural accompaniment of the abundant vitality and proliferation of new cells which characterize the growing organism.

Speech is a combination of the singing tone or vowel sounds produced by a column of air passing over the vocal cords and the whispering attitudes known as consonants produced by the various coordinations of the muscles of the mouth and throat. The endless experimental exploitation by the child of this vocal apparatus, in a stimulating context, which from our adult point of view we erroneously describe as imitation, is the basis of variable response from which is evolved in a remarkably short time his ability to speak and to thread his way verbally amid the maze of meanings among which the adults about him move with such apparent freedom and ease. That this ability is acquired in so short a time is only explicable by the fact that he is born into an environment of selected stimuli by which he is enabled to make innumerable short-cuts in a learning process in which his primitive forebears floundered for ages.

The limitations and the dangers of the symbol grow directly out of its usefulness. The very assistance which the word renders as a convenient handle to remoter objects and events and situations leads almost inevitably to a substitution of means for ends. Man gives a thing a name or

finds a word to mediate between an attitude and an act, and then uses the name as if it were the thing and the word as if it were the finality of response—forgetting that the name may be but the moment's rendering of the stimulus, that the word may be but the moment's embodiment of the response. Words come to be treated as the miser treats his coins, hoarded up and gloated over for their own sake, in disregard of the fact that they are but a medium of exchange, their value depending upon the concrete things they represent. Language is that portion of behavior which functions as a buffer between other parts of behavior, intermediary between the inaccessible tensions within the individual and the overt adjustments of social intercourse. It partakes of the character of both conduct and thought; it is less overt than what we call acts, but more overt than the tentative movements in the accessory muscles which we call thought. Its chief utility lies just in this ambiguous, this amphibious character. Theoretically a word alters its meaning every time it is used, since it is being employed to mediate factors in a situation different, to some extent, from any that has ever been encountered before; practically, the word becomes a compromise and a reducer of these differences to some common denominator of action. Here are both its great serviceability and its harmfulness in growth. In so far as this reduction of differences is subservient to the ends of an experimental expansion of experience, it makes for economy and efficiency in action; but when it becomes habituated to the point of functioning independently as a behavior-pattern, there is danger of the substitution of a system of abstract relations for the world of concrete individual facts.

It is clear, therefore, why language has been of so much value in building up what we call our intellectual life. A word can stand, not only for the extraorganic object or event remote in space or time, but also for the obscure intraorganic innervations and nascent movements for which an unriveted psychology has had no other descriptive terms than the vague popular terms feeling and thinking. It stands for these, and relates them in that total of overt activities which we

call the conduct of life. Language thus is a bridge between the inner citadel of the interoceptive and proprioceptive complex we call the self and that exteroceptive complex we call the outer world. Words are our 'innards' trying to find hands and feet; they are also the machinery by which we succeed in bringing an increasingly wider range of the environment under control, organizing it, in a very true sense, into the very substance of our selfhood.

A gesture, then, is an arrested act. A word is a substitute for a gesture. A thought is an incipient word. The image or idea or meaning or thought is but a name for the most-reduced of acts, the tentative partial performance which serves as a substitute for the deed in its overt entirety. Meaning is this indicative, this forward or backward referring, significance of such nascent responses. A meaning originally is a signal-fire, a notch cut, a mark made, a line drawn, to direct subsequent action. A monument, a cross, a badge, a label, a tally, a voucher, an autograph, an endorsement, a bill, credentials, insignia, a flag, an escutcheon, a password, a cipher, an epitaph has meaning because it records past or controls future behavior. The gradual reduction of these to the more abstract symbols of grammar, rhetoric, logic, mathematics, methodology is merely an accessory muscular refinement on the more fundamental motor adjustments.

12. THE ORGANIZATION OF A WORLD OF VALUES

We shall not have arrived at a comprehensive view of the evolution of behavior without calling attention again to a fact presupposed in all that has been said: namely, that all this elaboration of stimulus and ramification of response is ultimately and always for the sake of bringing fulfilment to certain inherited or acquired propensities. All this development of bilateral symmetry and the metameric form, this cephalization of the sense-organs and magnification of the brain, all the complications presupposed in the building up, through these, of spatial and temporal adjustments, and particularly of a world of incipient responses or symbolizations through the action of the accessory muscles—all this may

be said to be for the sake of finally reaching the distant object and its ingestion or manipulation in connection with food or sex. In other words, there is a final consummation of the means in the ends, of the instruments in the values, of life. This may be called the ultimate equation of a world of methods or means with a world of ideals or ends. In our human sphere it is the culmination of efficiency in culture, of science in art. In terms of the evolutionary process, it is a consummation of the function of the distance-receptors in that of the interoceptors and proprioceptors. The tactile-kinæsthetic imagery is the carrier of the meaning: an object, a situation, a world, seen, heard, smelled, is for the sake of a world touched, manipulated, enjoyed. It is the contact-values which are the goal of the pursuit of the distant object, and all of our economic and social institutions in human society are capable of interpretation from this point of view.

THE PRINCIPLES OF SERIAL AND COMPLETE RESPONSE AS APPLIED TO LEARNING

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The advantage of studying the behavior of animals in a maze, as a means of arriving at the factors involved in the formation of motor habits, is due largely to the ease with which those movements which are most detrimental to the habit-formation can be observed. Those movements are the ones occurring in the blind alleys; and it is their separation from the movements within the true path which facilitates their observation. Inasmuch as the learning of a maze depends upon the suppression, or elimination, of useless movements with the retention of the successful ones, and inasmuch as useless movements other than the ones into blind alleys, such as hesitations and retracings within the true path, disappear as a rule early in the course of the learning, the main problem of the observer is to account for the suppression of the blind-alley movements.

It is of capital importance, in attempting to solve this problem, whether one holds that the habit of running the maze is made up of a series of stimulations with a corresponding series of responses; or that it consists of a complete response to a complete set of conditions, the selection of the successful movements having been due to "the entire conformation of the organism together with the present more or less stimulating conditions." The former view is held by Dr. Harvey Carr among others, and underlies his exposition of the principles of selection in animal learning;¹ and the latter has been brought forward by Dr. Joseph Peterson in his presentation of completeness of response as an explanation principle in learning.² The following quotations, in which

¹ PSYCHOL. REV., 1914, 21, pp. 157-165.

² PSYCHOL. REV., 1916, 23, pp. 153-162.

Dr. Carr refers to the problem-box and Dr. Peterson to the maze, will summarize the respective views:

"The animal does not react to this complex situation as a unitary whole, as a single stimulus. He reacts to it selectively, and as a series of stimuli. There is a circular interaction between the sensory stimuli and the animal's movements. Each act modifies the stimulus in some respect, and the change of stimulus in turn modifies the act"¹ (Carr).

"Thus by an actual overlapping of many tendencies to respond in diverse ways the erroneous tendencies are directed into the successful ones, and the latter are strengthened by reinforcement. Without such overlapping of various impulses in the same general response, the inhibiting effects of the successful upon the unsuccessful or irrelevant tendencies are incomprehensible. . . . It is a mistake to look upon these tendencies as separate *acts* each complete in itself and occupying the whole arena for the time being. . . . The selectiveness, then, is due finally to the entire conformation of the organism together with the present more or less stimulating conditions; more immediately it is due to the cumulative effect of various incomplete partial responses"² (Peterson).

The serial-stimulation theory is the one generally held by students in this field of research, and most of them probably would assent to the 'circular-interaction' addendum to it. It has been recognized also that there is a less specific response which may be termed the set of the animal, or its attitude, in the maze-situation. This does not necessitate any other assumption than that the instinct of flight, which asserts itself upon the animal's introduction to a new situation, is no longer operative, and that entrance into the maze and obtaining food have become associated in the animal's nervous system. It is not necessary to assume, as Dr. Peterson does, that many of the stimuli which come into play in the course of running the maze are actually present when the animal is introduced into the maze, some of them directly and some of them indirectly by association.³

¹ *Op. cit.*, p. 157.

² *Op. cit.*, pp. 156, 157.

³ *Op. cit.*, p. 158.

The article of Dr. Peterson's does not expressly identify completeness of response with the sum-total of responses occurring in the maze. But the absence of any limitation, expressed or—so far as the writer can judge—implied, would indicate that these terms may be taken as synonymous. A similar observation may be made with reference to the term 'general response.' It would consequently seem that, according to this view, the responses which occur in the successful running of the maze are already at hand when the animal enters the maze, in virtue of a complete response to a complete situation; and successive stimulations may be said to call them forth only in the sense that they give them opportunity successively to manifest themselves.

It is undoubted that progress toward the solution of this problem has been made by the analytical method, with its recourse to the principle of a one-to-one relation between the series of stimuli and the series of responses, and to the principles of frequency and recency, even if it has been found that equal stress cannot be laid upon both these latter, nor upon the same one of them at all times. But those who have resorted to it have not claimed for it finality and perfection. In future inquiry, the analytical method may be adhered to, while new factors influencing selection may be sought; or this method may be considered to have yielded its modicum of explanation, and the student may turn to some new principle, whether it be completeness of response or some other; or, still again, investigation may be carried on according to more than one method. It is not the aim of this article to advocate exclusive reliance upon the analytical method, but rather to raise a doubt as to the availability of the principle of complete response to serve either as a substitute for, or as a supplement to, the analytical method.

Some of the difficulties connected with the principle of complete response will become apparent if we examine Dr. Peterson's account of the elimination of a blind alley; and these difficulties obtain whether the expression 'general response' denotes the tendencies pertaining to the maze as a whole, or those occurring only in a part of it. While a

rat is entering and leaving a blind alley, certain elements of the general response, according to this account, are still tending to drain into other alleys that have been recently passed. "Let us suppose that the correct path, *A*, has just been passed when the animal suddenly comes to the end of the cul-de-sac, *B*. The tendencies to respond to *A* are still surviving and now direct the impeded activity into this, the successful, path. If, on the other hand, the correct path had been chosen the first time the distracting impulses toward *B* would have become fainter and fainter as the animal proceeded into *A*, and would finally have faded away. The principle is not different when the complexity of the situation is increased. When the food is finally reached all the remaining delayed reactions, the tendencies, still persisting, to go into other alleys recently passed, are relaxed—the act as a whole is complete."¹

There are three difficulties. The first arises out of the statement that, when the animal suddenly comes to the end of the blind alley, *B*, it is the tendency to respond to the true path which impels it along that path, the implication being that this tendency draws the animal out of the blind alley before it impels it along the true path. It may be that Dr. Peterson did not intend to deny that any stimulation within the blind alley, such as the animal's butting into the end of it, has any effect in causing its retreat; but the fact that he could express himself so elliptically, leaving out any mention of stimulation within the blind alley, shows what stress he lays upon the conflict of tendencies as compared with the interplay of the organism and the environment. Furthermore, the animal on emerging from the blind alley is differently oriented toward the true path from the way it was before entering. It is not safe to assume that the solicitation of the true path in the later orientation will be as strong as it was in the earlier. Then, too, the movement into and out of the cul-de-sac may exert a modifying influence on the tendency to follow the true path, and there is no certainty that this modification will leave that tendency as strong as it was before the animal's entrance.

¹ *Op. cit.*, pp. 155, 156.

The second difficulty is to make clear how, when the animal is drawn back into the true path, the erroneous tendency is directed into the successful tendency as the result of a collision between them. There is no reference to any effect resulting from stimulation arising from the true path and leading to the orientation of the animal. It was observed by Carr and Watson¹ that rats which had already learned a maze, when placed at random in the maze, moved back and forth a few times, and then started in the right direction. The interpretation which they placed upon this behavior was that the animal had obtained a 'cue,' probably a kinæsthetic one, which directed it aright. It is possible, in the case Dr. Peterson considers, that the animal's retreat from the blind alley and entrance into the true path in a direction contrary to the correct one might result in a confusion, wherein exploration and orientation similar to that described by Carr and Watson might come about.

The third difficulty is that of understanding how, on the grounds alleged, the tendency along the true path, which was not strong enough to keep the animal from entering the blind alley, could gain sufficient strength, once the animal had been in and come out of the blind alley, to prevent its reëntering it. How the coming together of an erroneous tendency and a correct one could produce anything other than a resultant lying somewhere between them, much less result always in the reinforcement of the tendency in the true direction and never in the reinforcement of a tendency in a wrong direction, is not apparent. There is no help in appealing to the 'general response,' or to any redistribution in the general tension brought about by the overlapping of many responses, for it could not be proved that the bare entrance into and exit from a blind alley would produce such a change in this general response as to reinforce the tendency along the true path.

Some experiments performed by the writer, and to be reported in a forthcoming monograph, have a bearing upon the matter under consideration. Groups of rats were given

¹ Carr and Watson, 'Orientation in the White Rat,' *Jour. of Comp. Neur. and Psychol.*, 1908, 18, pp. 29, 44.

two or more trials in one maze, *E*, and then were transferred to another maze, *D*, which they learned to run without an error. After this they were brought back to maze *E*, and allowed to complete the learning of it. The mazes between which the transfers took place were of the same material, the same size, the same color, and were placed side by side, the only difference being that the true paths and the blind alleys, although of the same dimensions in both mazes, were differently arranged.

It was found that some of the rats on their return to the former maze ran it without an error four times in succession, thus meeting the criterion of mastery adopted in the experiment. Of seven rats, whose learning of *E* had been interrupted after two trials in order to learn *D*, three made a perfect score upon their return to *E*, although none of these three rats had made an errorless run upon either of their two former trials. Of seven rats, whose learning had been interrupted for the same purpose after four trials, one made a perfect score upon its return and another made but one error in five trials; and none of these had made an errorless run in any of its four previous trials. Of ten whose learning had been interrupted after eight trials, six made a perfect score and one a single error in five trials, among these being one rat which had made one errorless run and another which had made two errorless runs during their eight former trials. Of eight whose learning was interrupted after sixteen trials, two made a perfect score, one of these having made one errorless run and the other none.

If as a consequence of learning the *D* maze the rats had stored up the tendencies to be used in running that maze, these tendencies overlapping and being elicitable, many if not all, at one time, it is difficult to believe that these tendencies should not display themselves upon the return to the *E* maze, seeing that the elements composing the true paths and the blind alleys of the mazes are similar, and differ only in their arrangement. But they could not thus display themselves without interfering with the learning of the *E* maze. Yet we have seen that many of the rats upon their return traversed the maze without an error.

It is not believable, moreover, that the rats which made perfect scores upon their return to the *E* maze after having learned to run the *D* maze were enabled to run the *D* maze perfectly with the aid of certain abilities and to run the *E* maze perfectly with the aid of other abilities. Whatever would account for their running the *D* maze perfectly would account for their running the *E* maze perfectly; and, since it could not have been a set of tendencies constituting a complete response acquired in *D* which enabled them to run *E* perfectly, it is extremely doubtful whether it was such a set of tendencies which enabled them to so run *D*. Whatever were the abilities enabling the rats to run *E* perfectly after their return from *D*, those abilities were acquired while mastering *D*, for they became immediately effective upon their transfer back to *E*. But obviously those abilities could not consist in a set of tendencies used in running *D*, for they would not fit *E* and if called forth in *E* would only retard the learning. We cannot assume that while learning *D* a double process was going on, one part of which enabled the animal to learn *D* and another part of which was simultaneously preparing it to run *E* perfectly.

The only alternatives, as possible explanations of the general problem of the selection of the successful movements, are not, as Dr. Peterson seems to hold, that of completeness of response and that of a serial response to a succession of stimuli, in which these responses constitute separate acts "each complete in itself and occupying the whole arena for the time being." There is still another alternative: the stimulations may be serial in form, but before the effect of one stimulus is complete the next stimulus may begin to produce its effect. The rejection of the complete-response theory, therefore, does not mean the acceptance of the serial-response theory in the form which Dr. Peterson gives to it. It may be observed in this connection that, if the immediately preceding stimulus may begin to operate before the effects of the present one have ceased, a past response or past responses may also affect the present response through a modification which they have left in the nervous system of the organism,

thus influencing the organism toward one runway rather than another. But the assumption of effects due to the modification of the organism through past responses is a different assumption from that of the presence of a number of overlapping and inter-conflicting tendencies, some urging the organism in one direction and some in another.

While it would not be warrantable to assert that exclusive trust must be placed in the analytical method of dealing with this problem, nevertheless it is the method which has proved helpful thus far, and the only one that in the judgment of some investigators is applicable. There is one more factor to account for the selection of the successful movements, and to which attention should be drawn. It depends upon the fact that, on account of the structure of the maze, a successful movement must be made in its entirety—*i. e.*, through the whole length of the particular runway in which the movement is made,—every time the animal passes through the maze to the end, while an unsuccessful movement need not extend through the whole length of the blind alley.

The analytical method has dealt thus far mainly with the factors making for the selection of the successful movements and only indirectly for suppression of erroneous ones; but the factor to which attention is here drawn acts directly upon the erroneous movements, bringing about a gradual shortening of the distance to which the animal penetrates the blind alley. As was pointed out above, the whole series of stimuli do not necessarily affect the organism one after another with a sharp and rigid separation, but while one is operative the next may come into play. Consequently, if the animal, on its first entrance into a blind alley, received at the end of the alley the stimulus prompting its turning, it is not necessary on its next entrance that it should travel to the same spot before it comes under the influence of this stimulus.

The difference between this condition in the blind alley and a similar condition in the true path is that, no matter at what part of the true path the stimulus to turn into the next runway becomes operative, the animal must reach the end of the runway before giving way to it. This difference is due, as has been

already noted, to the construction and pattern of the maze, there being no physical barrier to the animals' turning within the blind alley, while there is such a barrier in the case of the runway forming part of the true path. The fact, then, that the various successful movements must be made in their entirety, through the whole length of the maze, but that erroneous movements need not be, taken with the further fact that there is a tendency constantly operative and reducing the length of the erroneous movements, as may be observed in the behavior of the animals,—these facts constitute an additional explanatory principle to be included in the analytical account of motor learning, the principle of the completeness of the single successful movement.

The writer hastens to add that this principle is not advanced as an entirely new one. Holmes, for example, says: "In behavior of the trial and error type, success is attained, not by a direct adaptive reaction, but by checking or reversing all reactions except the right one;¹ and Carr says: ". . . acts are selected or eliminated according to whether the sensory consequences tend to facilitate and intensify them on the one hand, or to disrupt and suppress them on the other."² Holmes, in the chapter from which the above quotation is taken, refers the advantage enjoyed by the successful movement to its connection with congenital modes of response which are adapted to secure the welfare of the organism; and Carr shows how the result of the final successful movement, which is entrance into the food-box, will ensure that the innervation connected with this movement will not be interrupted, but will reach completion.

It is not the purpose of the present writer to question such statements as the foregoing, the validity of which he accepts. It has been his intention merely to show that, in the case of the maze, inasmuch as the successful movements must be made through the entire length of the runway while the erroneous movements need not be, and there is a constant tendency making for the shortening of the erroneous move-

¹ Holmes, S. J., 'Studies in Animal Behavior,' p. 158.

² Carr, H., *op. cit.*, p. 162.

ments, the arcs involved in the successful movements must undergo greater innervation than any of the others. This, as already noted, seems to justify placing the principle of the completeness of the single successful movement on the same footing with frequency and recency as explanatory principles in maze-learning. It seems warrantable, moreover, to assume that this principle holds in all learning by the trial and error method.

THE INFLUENCE OF EXTRANEOUS CONTROLS IN THE LEARNING PROCESS

BY HARVEY CARR AND HELEN KOCH

University of Chicago

This paper attempts a preliminary comparison of the rate of learning when all possibility of error has been eliminated by means of some extraneous control, as opposed to the usual procedure of learning by the trial and error process. The two methods may be termed 'controlled' and 'free or undirected' learning respectively. The nature of the problem may be more adequately comprehended after a descriptive account of the apparatus and the mode of procedure.

A diagram of the problem box is given in Fig. 1. Its dimensions are $2\frac{3}{4} \times 5$ ft. It consists of a 10×13 in. food box *A*, an initial 3 ft. runway *C*, into which the animals are pushed through the door *B*, which is then closed, and two diverging paths *R* and *L* which finally merge into a common path *D* which in turn leads to the food box by the door *E*. These return paths can be closed as desired by sliding doors at *X* and *Y*.

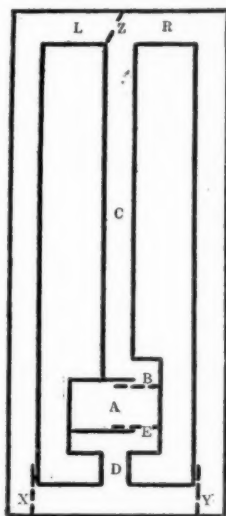


FIG. 1.—Diagram of problem box.

The problem to be mastered was the habit of alternating choices between the two paths *R* and *L* in the definite sequence of *R, L, R, L, R, L* for each day's test of 10 successive runs. In the controlled learning the animals were forced by the swinging door *Z* to choose according to the given sequence by alternately closing the entrance into each of the diverging paths *R* and *L*. But one path was open at a time; the

possibility of choice was eliminated; errors were impossible. The rat was forced to respond correctly in the desired sequence. In the test for free learning the door *Z* was removed and the rats were confronted with two paths at the end of alley *C*, of which only one gave access to the food box. One error was thus possible in each trial, and the animals were compelled to learn to eliminate these errors and to choose correctly,—to learn to select the proper sequence out of two possibilities.

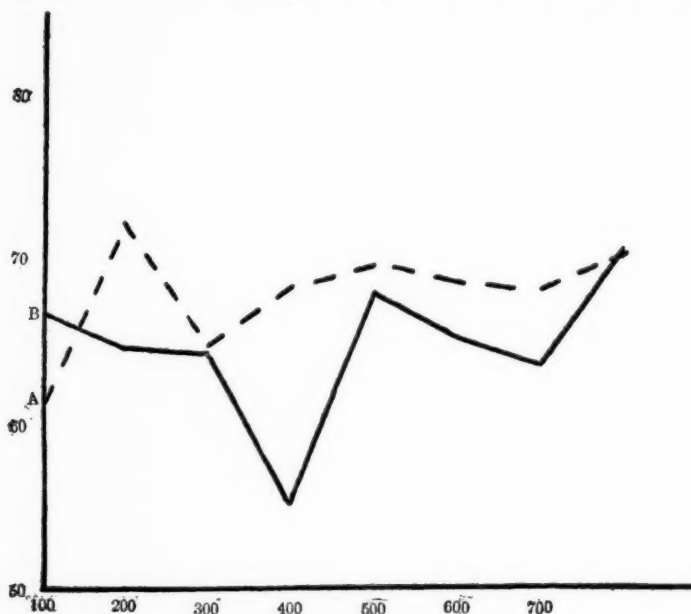


FIG. 2. Graphs representing percentage of correct choices.

The group employed in the free learning consisted of 12 rats, and they were given 10 successive trials daily. The record of this group is represented by graph *A*, Fig. 2. The values represented are the percentages of successful choices for successive 100 trials. Since but two opportunities of choice are given, it is obvious that the initial record of a group of rats will approximate 50 per cent. if the choices are distributed according to chance. As the problem is mastered the curve will rise from 50 per cent. to 100 per cent.

The controlled group contained 13 rats, and the previous conditions were duplicated in every respect with the exception that the responses were controlled by means of the swinging door Z. With this group training and test series were necessary. In the training series the responses were controlled as previously described. In order to determine the degree of mastery resulting from this mode of training, a test series was interpolated at regular intervals, in which the door Z was removed and the animal allowed to choose between the two paths as in the 'free' procedure. A test series was given every 5th day, or 10 test trials were given after each 40 training trials. The records representing the mastery of the problem with this procedure are thus based upon these test trials alone. The records for this group are represented by graph B, Fig. 2, in which the values are the percentages of correct choices for the 20 test trials given for each successive 100 trials. The two graphs thus represent the relative efficiency of 100 free trials as compared with 80 controlled and 20 free trials.

It is evident from an inspection of these graphs that neither group is likely to master the problem with any high degree of proficiency within 2,000-3,000 trials. Since we wished to study the relative effect of various combinations of free and controlled trials, it was soon apparent that the alternation problem was unsuited to our purpose because of the extended time necessary for its mastery. Consequently we decided to discontinue the experiment in its present form and employ some simpler types of problematical situation like the maze and latch box.

While the group graphs give but little indication of any possibility of ultimate mastery, yet an inspection of the individual records reveals the fact that seven of the 25 rats made considerable progress and gave every indication of finally attaining a high degree of proficiency. Each of these seven animals attained a record of 80 per cent. or better; three made records of 90 per cent. or better. Two other animals started with very high records but gradually decreased their scores throughout the experimentation. The

remaining 16 rats rapidly adjusted themselves to the problem at first but soon fell into a fixed mode of response and made no further progress.

There is but little difference between the records of the two groups, but such as it is favors the group with the free mode of learning. Of the seven animals that attained a high percentage of correct choices and gave evidence of an ability to master the problem, four belonged to the group whose learning was undirected. The values for the group given the free mode of learning are the higher throughout with the exception of the initial value. This initial record constitutes no valid exception to the superiority of this group, however, as this value is based upon the initial 100 trials, while the corresponding value for the group whose learning was controlled is based upon 20 trials after the animals had been subjected to 80 training trials. By computing the initial record of the group with the free mode of learning at a corresponding period in their training, a percentage of 72 is secured. With this correction, graph *A* represents the higher values throughout with the exception of the final value where the two records are approximately identical.

Although the 80 controlled trials were not as effective as a corresponding number of trials where choice was allowed, yet one can not maintain that these controlled runs were wholly without effect. It is hardly probable that the three animals of this group that attained a high degree of mastery of the problem would have been able to do so on the basis of the 160 free trials alone. Moreover the initial record of the group is 66.6 per cent., which is considerably above what would have been secured if these choices had been distributed according to chance. Two of the group made initial records of 90 per cent., and one a perfect record of 100 per cent.; moreover, two of these animals consistently made high scores throughout the experiment. Evidently the initial values for some rats at least have been effectively influenced by the controlled runs of the training series.

Our results indicate that the given proportion and distribution of the controlled trials was effective in mastering

the alternation problem especially during the early stages of the learning process, but that the efficiency of these runs is not as great as that of a corresponding number of free trials. The experiment can not, however, be regarded as a decisive test of the relative values of the two modes of procedure.

For other purposes the experiment with the group given free choices was continued for a total of 1,250 trials for each rat. At this point the group was given controlled and free trials on alternate days for 20 days,—a total of 200 controlled and free trials for each animal. The test was designed to determine the efficacy of an extraneous control in the later stages of mastery. The introduction of these 100 controlled runs produced no noticeable effect upon the course of the group graph. Neither was any effect apparent in the majority of the individual records. The scores of three rats may have been slightly bettered, and those of two slightly lowered. In general those animals that had developed strong position habits were certainly not affected. Those individuals that had developed no fixed scheme of response may have been affected to some extent. The new mode of procedure seemed to alter their normal mode of response, and this induced variation might be either advantageous or detrimental.

The influence of an external control in the learning process constitutes a problem of which 'learning by being put through' is a special case. Thorndike first employed this method and found that it was ineffective. Subsequent experimentation, with a few exceptions, has confirmed his results. In Thorndike's experiment, the animals were relatively passive,—the movements being both initiated and directed by the experimenter. Our conditions differ from his in two respects: our animals initiated their movements, and their activity was controlled by mechanical means. Thorndike concluded that the ineffectiveness of his method was due to the absence of a motor impulse to be associated with the sensory situation. He intimated that learning would have occurred if the animals had actively participated in the reaction and initiated their movements. Our results indicate that the presence of a

motor impulse does not necessarily constitute a very favorable condition of learning.

Our procedure is more nearly identical with certain educational methods of instruction in vogue in teaching such acts of skill as writing and dancing in which the subject is forcefully guided by manual means.

Our problem also raises the theoretical question of the value of errors or mistakes in the learning process,—a question which has never received any extended discussion in the literature on learning. It is usually assumed that errors have a value in an adaptive problem in that they aid the quick discovery of the proper means of solution. The greater the amount of random movements,—the greater the exploration, the sooner will the successful act be discovered. After the solution has been discovered, however, general opinion would probably contend that errors are no longer useful but actually detrimental, inasmuch as they represent tendencies to action which must be slowly eliminated by repeated trials.

Our provisional results do not support the above assumption that errors are invariably detrimental in the process of fixation; rather, they indicate that the process of fixing an association may be hastened by the inhibition of wrong responses. Given any problematical situation consisting of two alternatives, it is possible that an animal may not always react to these paths as two separate objects which have no relation to each other; they *may at times* react to two diverging paths as a unitary situation consisting of two related aspects. In the latter event it is possible that the correct choice is effective in part because the antagonistic tendency was inhibited, as well as because the animal performed the proper act. In such a situation, rejection and selection are relative terms and the process of rejection will emphasize the act of selection. The mere *doing* of a sequence of acts will tend to associate them to some extent, but their fixation will be further facilitated by the process of selection and rejection. The situation may be envisaged more easily in the problem of memorizing where mistakes when noted may be much more effective in establishing the desired association than a con-

siderable number of repetitions. The conception may also explain in part the effectiveness of an active attitude in memorizing; in the passive attitude the subject experiences the items to be associated as they are presented; with the active attitude the subject may either review the syllables previously exposed or attempt to anticipate the coming ones. In either case errors may be made and subsequently noted, and the effectiveness of the method may be due in part to this process of comparison of the proper associates as contrasted with the erroneous suggestions.

In this paper we wish to refrain from any general conclusions. Owing to the indecisiveness of our experiment, the factual results must be accepted with caution. Granted their validity, however, it is possible that an external control may be effective in one type of problem and not in another. The effectiveness of such a control may vary with the degree to which it is used during the learning process. It is also conceivable that an external control may be very effective when utilized only at certain critical stages in the development of a habit system. Miss Koch is now engaged in investigating these questions with the use of the maze and the latch box problem. She also plans to use human subjects with a pencil maze and employ various modes of control including that of verbal directions. This work has progressed to the point where we are able to say that a limited amount of control introduced at a certain stage of the learning is extremely effective in the mastery of the maze problem.

MULTIPLE CHOICE EXPERIMENT APPLIED TO SCHOOL CHILDREN¹

BY ELEANOR ROWLAND WEMBRIDGE AND PRISCILLA GABEL

The following test were designed as an application to human beings of the multiple choice method of testing, suggested by experiments which Major Yerkes once tried on pigs, crows and monkeys. In his account of his experiments,² he announced his intention of trying them upon human beings at a later date. But in the absence of any published data, we devised a choice experiment which seemed to embody the same principles as those employed in the animal work. Our experiment consisted merely in the choice of certain cards, instead of the choice of food boxes, as in the original experiments. These card-choices increased in difficulty, just as the series increased in difficulty in the Yerkes experiments. After applying the series to 100 children in the public schools, the resulting figures were correlated with the results of Binet intelligence tests. The Pearson coefficient was used.

It will be remembered that the Multiple Choice experiment was devised by Yerkes in order that a series of problems ranging from the simple to the complex might be applied to organisms of different types and conditions and at different stages of development. In the accounts published, Yerkes tried four different choices on three different types of animal. The animal was placed before a series of open mechanisms, and was induced to enter and to learn their relative positions, because he found that food was only in the correctly chosen

¹ From the Reed College Psychological Laboratory.

² Yerkes, R. M., and Coburn, C. A. 'A Study of the Behavior of the Pig, *Sus Scrofa*, by the Multiple Choice Method,' *J. of An. Beh.*, 1915, 5, 185-225.

Coburn, C. A., and Yerkes, R. M., 'A Study of the Behavior of the Crow, *Corvus Americanus* Aud., by the Multiple Choice Method,' *ibid.*, 75-114.

Yerkes, R. M., 'The Mental Life of Monkeys and Apes: A Study of Ideational Behavior,' *Beh. Monog.*, 1916, 3, No. 12, pp. 145.

box. He was taught to avoid the wrong choice, because food was not in any of the other boxes. He was entrapped by the door when he entered the wrong mechanism, and therefore unpleasant associations were established with the wrong choice.

The tests used were the following:

1. First mechanism to left of subject
2. Second mechanism to right of subject
3. First mechanism to left and right alternately
4. Middle of group

The animal tests of Yerkes are fully described in his own publications cited above, and will not be discussed further here except to say that the mental development of the three types of animal upon which he experimented was compared by their ability to grasp and remember the relation of the food boxes to each other, so that eventually the right box might always be chosen by its position.

Obviously in giving the same type of tests to human beings, some different variety of technique had to be devised. In the first place, the series had to be much enlarged. To this end, a series of fifteen choices was planned, of which the first four were the same as in the animal series of Yerkes.

Secondly, there was no necessity of rewards and punishments in order to make the subject take the tests, and wish to excel in them. Therefore, the choice was made from relative positions of cards arranged on a table before the subject, from which he was asked to select the 'right one.' His success in discovering which was the right one, and interpreting the choice scheme, was stimulus enough for interest, and for the attempt to do his best. The comparison was made of the number of trials needed to select the right card, rather than of length of time taken in choosing, for the length of time taken seemed to be more a temperamental factor, involving greater or less timidity, greater or less effort expended, etc., rather than difference in ability to perceive relationships. Our first problem was to increase the series of tests, from the four given by Yerkes, to a longer series, and to be certain that they increased gradually in difficulty.

Our second problem was to apply this graded series to enough subjects to make our figures in any way reliable. We devised eleven additional tests, and in order to place them in a series of increasing difficulty, we applied our series of fifteen tests to sixty children in the public schools. A norm was established by giving this list of fifteen tests, and forming a curve of increasing difficulty, as judged by the increased number of failures in solving the various tests.

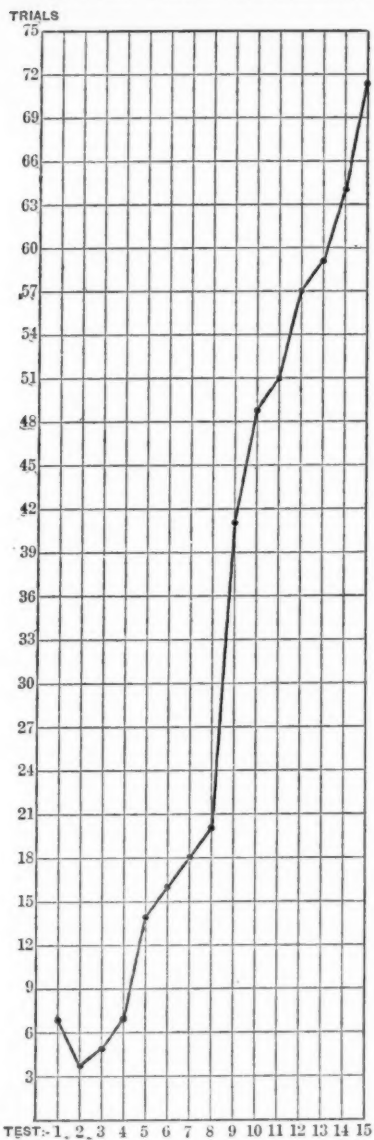
The list that was finally adopted was the following:

- 1-4. Identical with Yerkes tests.
5. Second card from each end alternately.
6. Third card from right of subject.
7. First and third; second and fourth cards from left alternately.
8. Second and fifth cards from left, alternately.
9. First card to right of middle (even number in series).
10. Fourth card from left and third card from right, alternately.
11. Fifth card from right.
12. Fifth card from left.
13. First card to left of middle (odd number in series).
14. Second card from left of the series and the middle card.
15. Third card from right, and fifth from the left; and fifth card from the right, and third from the left alternately.

The relative difficulty of the scale is shown by the curve plotted of the number of failures in each test. The whole scale was given to each child, and failure to solve the test in twenty-five trials was counted as failure.

The tests were given in the following manner. The series of nine cards (corresponding to the nine food boxes) was laid on the table. When the child entered the room, the experimenter addressed him, "I have some puzzles to show you. [Calling the tests 'puzzles' immediately aroused his interest.] Please choose one of these cards." Since the first choice was purely random, and probably wrong, he was asked to choose again, until the correct card was chosen.

The random choices until the card was first correctly



Graph showing relative difficulty of multiple choice tests judged by the number of failures.

chosen, were not counted. When once it had been selected, it was pushed a little above the others, in order to make its relative position more apparent to the subject. Then the series was changed to five cards, fourteen cards, ten cards and so on, and the child was asked to choose again, until he recognized its relation to the series, and chose correctly every time. The series was never longer than 15 cards. When it was certain that the child had solved a certain test, the one next in order was given, and the number of trials necessary to solve any of the tests after the first correct guess, was tallied by the experimenter. If in twenty-five trials the subject failed to solve a test, it was called a failure, and the following test was given. No reward was necessary for children. It was sufficient to tell them that their choice was correct, and their interest was so thorough, that they made every effort in most cases to make a good record.

In figuring the results an average was taken of the number of trials necessary for each subject to solve the fifteen puzzles. Each failure was counted arbitrarily as 30, whether the child actually tried thirty times or not. If it was evident that he could never solve the test, he was not wearied by random trials, but counted as 30. An average was then taken of the combined averages of the children in each age group, with the following result:

| Average no. of Trials in | | | | |
|--------------------------|-------------|-------------|--------------|--------------|
| 7-yr. Group | 8-yr. Group | 9-yr. Group | 10-yr. Group | 11-yr. Group |
| 18.6 | 17.0 | 14.5 | 14.2 | 12.4 |

Each of the hundred children who had taken the Multiple Choice test, was then given the Stanford revision of the Binet tests. The mental ages of the children, according to the Stanford revision, were then correlated with the average number of trials required by each child to solve the series of Multiple-Choice tests. The figure obtained by the Pearson coefficients was $r = .48$, which is significant as showing a high correlation between the standard mental test already in use, and this Multiple-Choice card test.

The correspondence seemed however to be least with the children of foreign parentage, who had trouble with the

language problems in the Binet tests. There were too few of these foreign children to make their correlation figure statistically significant from these experiments. We would however suggest that such a card test might prove valuable, as have some of the army psychological tests devised for illiterates, where the language factor makes some of the standard tests impossible to give. This might be tried out further by experimenters who have access to larger numbers of children of foreign extraction than did we.

PRACTICE EFFECTS IN A TARGET TEST—A COMPARATIVE STUDY OF GROUPS VARYING IN INTELLIGENCE

BY BUFORD JOHNSON

Bureau of Educational Experiments

This investigation was made for the study of the comparative practice effects in a motor test given to groups varying in intelligence as measured by standardized scales. It has been generally supposed that those ranking as mental defectives would show less improvement by practice, be less able to profit by experience than those within the normal range. Very few studies have been made to obtain actual facts in given trials.

Ordahl and Ordahl¹ reported on the differences between levels of intelligence in feeble-minded. They took 30 cases of typical feeble-minded persons ranging in chronological age from fifteen to thirty-five, forming three groups of 10 each, with mental age respectively 6 years, 8 years, and 10 years, according to Binet ranking. They found that in learning a series of visual-motor coördinations, there was a marked difference in speed and accuracy for the three groups, ability increasing with mental age. Differences in initial ability were just as distinct, and the inability to establish control for the six year group would invalidate their results in a comparative learning test, though all groups have practice curves that are similar in form to those for normals.

Colvin² paired five normal subjects with five subnormal, using the Binet tests as a standard, and gave practice work in cancellation of A's. He reported that the normal child made greater improvement with less fluctuations. Complete data on this have not been available.

¹ Ordahl and Ordahl, 'Qualitative Differences Between Levels of Intelligence in Feeble-minded Children,' *Jour. Psycho-Asthenics, Monog. Sup.* 1., No. 2, p. 33.

² Proceedings of the American Psychological Association, *Psychol. Bull.*, 1915, 12, p. 67.

Kuhlman¹ made an early study of an aiming test with 9 subjects—6 imbeciles, 3 morons. He reports "their practice curve goes down after the second week through a decreasing interest in the work and rises again when that interest is artificially aroused. The regularity of their throw increases with practice. They miss the whole target less after a while and also hit the center less after a while than at the beginning."

Woodrow² made an investigation upon practice effects and transference in normal and feeble-minded children, comparing the feeble-minded of average mental age nine, and average chronological age fourteen, with normal children of nine years of age. He found that the feeble-minded children improve with practice the same as normal children of like mental age, and that there is no significant difference in the amount of transference. His most striking conclusion is that there is an absence of correlation between capacity to learn and capacity to grow; that when practice is continued sufficiently long, there will be improvement due to growth rather than practice, and that then the normal children would outstrip the feeble-minded children of the same mental age. This conclusion is presented, however, as indicated by the data, and not established.

The subjects for the present investigation were inmates of the New York State Reformatory for Women at Bedford Hills, New York. They were selected according to mental age as determined by the Stanford Revision of the Binet and the Yerkes-Bridges point scale. Three groups of five each were formed, one group representing the upper mental level of the inmates; another the median reformatory type, who are just across the border-line in the inferior group of normals; a third, the lowest or clearly feeble-minded class. Records of two of the highest group are omitted because of defective vision and failure to obtain glasses for correction. One of the low group, as originally formed, moved to another cottage, and her record for the short period was discarded.

¹ 'Experimental Studies in Mental Deficiency,' *Amer. Jour. of Psychol.*, 1904, 15, p. 413.

² 'Practice and Transference in Normal and Feeble-minded Children.' Part I.: Practice,' *Jour. Ed. Psychol.*, 1917, 8, 94.

Table I. shows the chronological age together with the mental ranking of each subject.

TABLE I

| Group | Individual | Chron. Age | | | Stan. Rev. | | | Verkes-Bridges | |
|--------|--------------|------------|------|------|------------|------|-------|----------------|-------|
| | | | | | Men. Age | | I. Q. | Score | C. I. |
| | | Yrs. | Mos. | Days | Yrs. | Mos. | | | |
| High | a. | 18 | 9 | 9 | 15 | 3 | 95 | 92 | 1.05 |
| | b. | 21 | 6 | 9 | 15 | 4 | 96 | 88 | 1.00 |
| | c. | 28 | 5 | 6 | 17 | 2 | 107 | 95 | 1.08 |
| | Average..... | 22 | 10 | 28 | 15 | 11 | 99.3 | 91.7 | 1.04 |
| Median | d. | 20 | 11 | 27 | 11 | 10 | 74 | 78 | 0.89 |
| | m. | 16 | 7 | 26 | 10 | 6 | 66 | 67 | 0.80 |
| | n. | 24 | 3 | 13 | 11 | 3 | 70 | 73 | 0.83 |
| | o. | 19 | 5 | 29 | 12 | 9 | 80 | 75 | 0.85 |
| | p. | 16 | 9 | 21 | 11 | 9 | 73 | 75 | 0.87 |
| | Average..... | 19 | 7 | 23.2 | 11 | 7.4 | 72.6 | 73.6 | .848 |
| Low | w. | 17 | 0 | 25 | 8 | 9 | 55 | 54 | 0.63 |
| | x. | 21 | 4 | 25 | 10 | 6 | 66 | 61 | 0.69 |
| | y. | 17 | 10 | 27 | 8 | 2 | 51 | 54 | 0.51 |
| | z. | 24 | 4 | 25 | 8 | 2 | 51 | 58 | 0.66 |
| | Average..... | 19 | 5 | 24 | 8 | 10.8 | 55.8 | 54.5 | .623 |

The individual variability and, in some cases, slight improvement raise a question of the significance of this form of test as a type of learning ability. Is ability to throw so nearly a learned process with adults that a few trials in adjusting this ability to a new situation give a relatively final capacity? To make a more intensive study of this test as one of learning ability, the scores of two other subjects, both rated efficient stenographers, are given. These subjects were given more total trials and more throws during a practice period.

APPARATUS AND PROCEDURE

The practice given was in an aiming test. A target board in the form of a circular piece of cork composition 2 feet in diameter, was placed in a vertical position upon a wooden base. This base was fastened between two uprights, along which it could be slid so that the target could be adjusted to the height of the subject. On this board three concentric circles were described in black ink, the lines of demarcation being $3/16$ of an inch wide. The circumference of the circular board formed the fourth circle. The diameters of these circles were respectively 6, 12, 18, and 24 inches.

Bull's-eye in the center was a circle $1\frac{1}{4}$ inches in diameter made solid black. The darts used were those manufactured by the Apex Dart Company, having a wooden handle $4\frac{1}{2}$ inches long, and a metal point $1\frac{1}{4}$ inches long. A length of 2 inches on the other end ~~was~~ feathered. An objection to the darts used was the difference in weight ranging from 2.6 to 2.13 ounces, apothecaries' weight. The only means used to meet this was the selection of paired groups of darts, each pair being approximately the same weight. A series of darts of the weight range mentioned was used, and if the point to one was bent, another of the same approximate weight was substituted. In this manner each subject had the same supply of darts, though the varying weight would doubtless interfere in a fine measure of increasing skill in the test.

Preliminary trials indicate 10 feet as the best distance from the board for adults, and this was adopted. Each subject stood at this distance from the plane of the board, and in such a position that his right arm when stretched horizontally in front of him was on a level with the bull's-eye. Similar position was taken for the left hand throwing. No special position of the feet was required except that one foot must touch the line and must not go over it, nor was there a controlled method of throwing the darts other than that in every case a straight overhand throw of varying trajectories and initial motions was used, and the chief difference observed was the manner of holding the dart. Some grasp it near the point, others much nearer the feathered end. One subject's manner of holding it or the motion given in the hurling, often made the dart turn completely over in the air and then stick into the board. At other times a partial turn was made, and the feathered end hit the board. She showed poor ability, especially with the left hand, and it was noticed that this peculiarity in throwing occurred more frequently with her left hand.

The subjects were brought to the laboratory in groups of five. Only the performer and observer were in the room where the apparatus was set up during the practice work.

This was given daily at the same time of day—between 10 and 12 in the morning—for four weeks. There were a few special cases, when a subject could not come in the morning, and the practice was given in the afternoon. There were necessarily many absences even in such a small group during so long a period. The effort was then made to give each one the same amount of practice, and the intervals between periods were noted. Two girls, X and Z, said that in an amusement park at Coney Island they had several times engaged in throwing at a target with darts like the ones used.

A comparison of the average scores for the first day's practice shows that the median and low groups have approximately the same initial ability, while the high group is superior with the right hand, but not with the left. So many factors enter into the first trial, especially the attitude of those who are rather unstable emotionally, as many of these are, that the results are not valid indications of initial ability.

A control group for each class was planned, and one day's practice given to seven. Varying causes prevented more than four subjects of this group taking the end practice. Since a single trial is not considered representative in a motor test, more practice periods should have been given at the beginning and end for valid measurement with control group. The very small number for end practice does not give sufficient representation in each group for a comparative measure. For these reasons no attempt is made at evaluation of practice effects as compared with a control group.

QUANTITATIVE RESULTS

Group comparisons based upon the average scores are made. The comparisons of the initial and final records, the first five and the last five trials, initial score and total average score are also made.

Distribution of Scores.—Tables II., III., and IV. show the individual scores for each practice period and the average daily score for each group, also the individual averages for all trials. Where a small number is placed, it indicates an interval of that many days between practice periods. The

graphs show for the median group the usual form of learning curve. This group has decidedly less variability than the others. There is a similarity in the form of curves for the low and high groups, with marked fluctuations. The downward slope in fourth week for the low group might be taken as an indication of less interest or need of greater stimulation after so long a period of practice. For the high group this slump even below initial score, occurs earlier. These interferences may be due to the great degree of variability, or

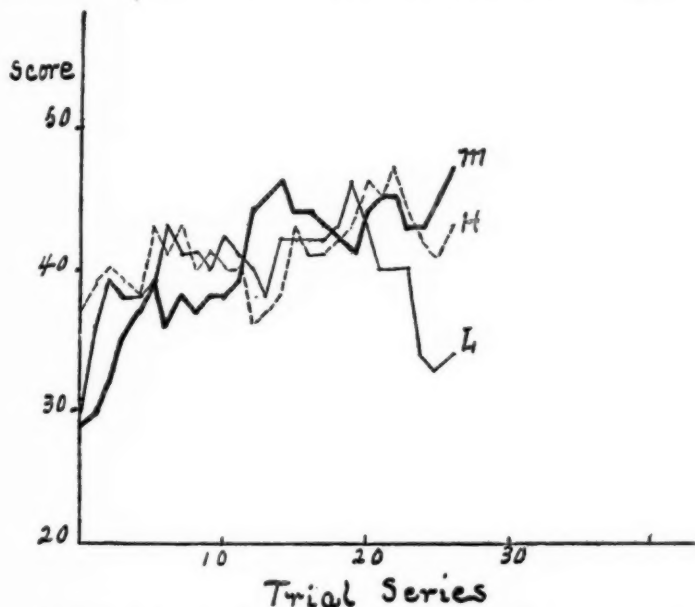


FIG. 1. Group Learning Curves, based on averages of each three successive daily scores made in 10 throws with Right hand. *M* is median, *H* is high, *L* is low.

perhaps to some emotional attitude, a factor to be discussed later.

Individual and group averages for left hand show superiority of right hand. In points gained the median group again leads in comparing first with final trial and first five with last five trials. They also have higher average score

by 1.4 points and are much more closely grouped together. The individuals maintained the same relative ranking as in R.H. scores with two exceptions, one in the low group and one in the median group.

The comparison of the curves for the low and high groups with that of the median indicates other differentiating factors than mental age as explanatory of the differences found.

The three subjects composing the high group show great individual inconsistency. *A*, who was decidedly the best thrower of all who have practiced in this test, making the

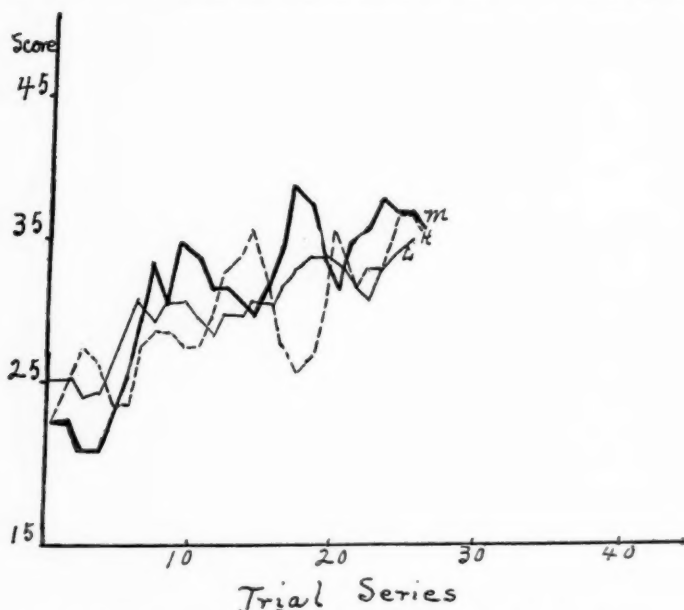


FIG. 2. Group Learning Curves, based on averages of each 3 successive daily scores with left hand. *M* is median, *H* is high, *L* is low.

highest final average and maximum score, was of a very emotional temperament, easily excited and dominated by moods. Her distribution curve of scores, shown in Fig. 3, is dominated by steeples, and her record was an overbalance in her small group. *C*, of highest mental ranking, showed

very poor motor coördination, seemed of the physically apathetic type, never having been interested in athletics or handicraft. Her final average was but a tenth of a point ahead of that of *Y*, the lowest in mental status. The graph for *C* shows the same predominance of marked fluctuations,

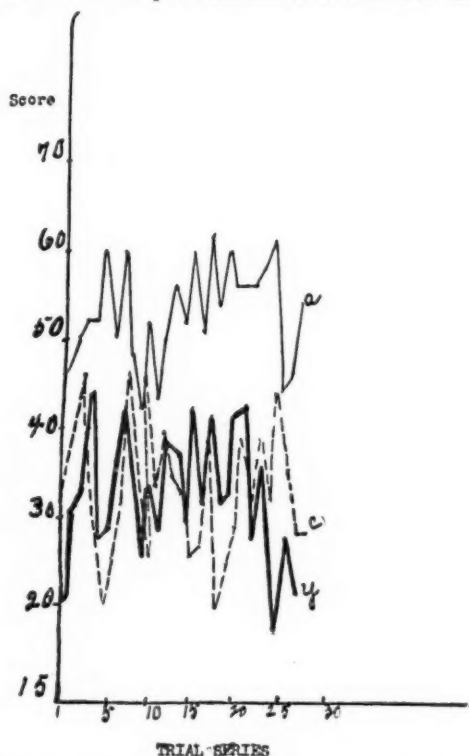


FIG. 3. Individual Learning Curves, based on daily averages of 10 throws with right hand. *A* and *C* belong to high group. *Y* belongs to low group.

but here they seemed to be caused by motor rather than emotional instability. Chance seemed the determining factor. There was little evidence of the development of a muscular memory in coördination with the fixation of the bull's-eye. She could never tell why nor when the dart would make a turn in the air. The arm muscles were held tense and the

attempt made to throw with the shoulder. With Y, of lowest intelligence ranking, the variability and final form of curve seem indicative of a lack of interest. Greater stimulation than the competitive spirit or observation of success would be necessary.

TABLE II
INDIVIDUAL SCORES OF LOW GROUP

| Practice Period | Right Hand | | | | Daily Average | Left Hand | | | | Daily Average |
|-----------------|-----------------|------|------|-------|---------------|-----------------|------|------|-------|---------------|
| | w | x | y | z | | w | x | y | z | |
| 1. | 7 | 46 | 20 | 33 | 27 | 20 | 30 | 16 | 33 | 25 |
| 2. | 30 | 37 | 30 | 52 | 37 | 17 | 36 | 14 | 32 | 25 |
| 3 | 39 | 39 | 33 | 51 | 41 | 25 | 15 | 21 | 35 | 24 |
| 4. | 21 | 38 | 43 | 40 | 36 | 24 | 17 | 13 | 42 | 24 |
| 5. | 33 | 36 | 27 | 35 | 33 | 28 | 24 | 18 | 25 | 24 |
| 6. | 43 | 41 | 28 | 54 | 42 | 34 | 28 | 36 | 31 | 31 |
| 7. | 42 | 32 | 36 | 48 | 40 | 28 | 27 | 24 | 33 | 28 |
| 8. | 41 | 37 | 42 | 55 | 44 | 30 | 24 | 24 | 44 | 30 |
| 9. | 49 | 33 | 25 | 42 | 37 | 40 | 15 | 24 | 34 | 28 |
| 10. | 35 | 33 | 33 | 54 | 39 | 38 | 9 | 28 | 50 | 31 |
| 11. | 50 | 42 | 28 | 47 | 42 | 34 | 26 | 24 | 39 | 30 |
| 12. | 46 | 40 | 38 | 48 | 43 | 33 | 17 | 22 | 32 | 26 |
| 13. | 44 | 22 | 37 | 28 | 33 | 31 | 21 | 28 | 31 | 28 |
| 14. | 49 | 41 | 29 | 48 | 42 | 32 | 19 | 42 | 33 | 32 |
| 15. | 42 | 22 | 42 | 42 | 37 | 34 | 15 | 15 | 48 | 28 |
| 16. | 54 | 40 | 31 | 47 | 43 | 43 | 9 | 35 | 35 | 30 |
| 17. | 41 | 28 | 41 | 56 | 42 | 25 | 17 | 34 | 51 | 32 |
| 18. | 35 | 25 | 31 | 58 | 37 | 28 | 12 | 27 | 41 | 27 |
| 19. | 46 | 40 | 32 | 62 | 45 | 43 | 25 | 24 | 47 | 34 |
| 20. | 48 | 38 | 41 | 48 | 44 | 35 | 16 | 43 | 40 | 34 |
| 21. | 46 | 44 | 42 | 50 | 46 | 36 | 19 | 27 | 46 | 32 |
| 22. | 40 ¹ | 29 | 27 | 46 | 36 | 32 ¹ | 22 | 34 | 40 | 32 |
| 23. | 40 | 22 | 35 | 42 | 35 | 37 | 16 | 26 | 50 | 32 |
| 24. | 54 | 42 | 38 | 52 | 47 | 34 | 19 | 25 | 42 | 30 |
| 25. | 38 | 35 | 17 | 44 | 34 | 31 | 15 | 26 | 41 | 28 |
| 26. | 33 | 29 | 27 | 58 | 37 | 53 | 17 | 38 | 48 | 39 |
| 27. | 34 | 24 | 21 | 48 | 32 | 41 | 19 | 21 | 46 | 32 |
| Total | 1,080 | 935 | 874 | 1,288 | 1,051 | 886 | 529 | 711 | 1,069 | 796 |
| Average . . . | 40 | 34.6 | 32.4 | 47.7 | 38.9 | 32.8 | 19.6 | 26.3 | 39.6 | 29.5 |

$$\sigma = 5.903, \sigma M_1 = 2.9515.$$

$$\sigma = 7.435, \sigma M_1 = 3.7175.$$

The low group included the two who spoke of previous practice, and who were also very alert physically. Psychographs showing their ranking in the entire series of tests given them at the laboratory, showed a marked contrast in ability for motor tests and for other tests primarily classed as mental.

A comparison of initial and final ability may be made from Table V. The median group shows greatest improve-

TABLE III
INDIVIDUAL SCORES OF MEDIAN GROUP

| Practice Period | Right Hand | | | | | Daily Average | Left Hand | | | | | Daily Average |
|-----------------|------------|----------|----------|----------|----------|---------------|-----------|----------|----------|----------|----------|---------------|
| | <i>l</i> | <i>m</i> | <i>n</i> | <i>o</i> | <i>p</i> | | <i>l</i> | <i>m</i> | <i>n</i> | <i>o</i> | <i>p</i> | |
| 1. | 33 | 32 | 30 | 22 | 19 | 27 | 22 | 20 | 27 | 24 | 14 | 21 |
| 2. | 27 | 18 | 44 | 30 | 52 | 34 | 9 | 25 | 36 | 21 | 29 | 24 |
| 3. | 27 | 43 | 33 | 12 | 37 | 30 | 14 | 20 | 20 | 26 | 20 | 20 |
| 4. | 31 | 33 | 46 | 25 | 31 | 33 | 22 | 14 | 19 | 15 | 16 | 17 |
| 5. | 36 | 48 | 48 | 32 | 40 | 41 | 26 | 15 | 21 | 23 | 27 | 22 |
| 6. | 38 | 42 | 42 | 35 | 23 | 36 | 35 | 19 | 25 | 31 | 31 | 28 |
| 7. | 45 | 40 | 45 | 33 | 42 | 41 | 30 | 6 | 17 | 38 | 35 | 25 |
| 8. | 38 | 38 | 30 | 25 | 30 | 32 | 37 | 19 | 31 | 48 | 35 | 34 |
| 9. | 42 | 40 | 58 | 39 | 26 | 41 | 35 | 32 | 32 | 58 | 32 | 41 |
| 10. | 38 | 37 | 40 | 41 | 30 | 37 | 32 | 23 | 43 | 42 | 23 | 33 |
| 11. | 47 | 44 | 39 | 33 | 23 | 37 | 39 | 22 | 23 | 41 | 20 | 29 |
| 12. | 43 | 39 | 48 | 46 | 27 | 41 | 33 | 52 | 29 | 44 | 24 | 36 |
| 13. | 39 | 52 | 28 | 32 | 49 | 40 | 30 | 27 | 23 | 39 | 26 | 29 |
| 14. | 39 | 52 | 50 | 47 | 48 | 47 | 29 | 31 | 34 | 22 | 31 | 29 |
| 15. | 52 | 42 | 50 | 43 | 42 | 46 | 52 | 19 | 31 | 37 | 25 | 33 |
| 16. | 47 | 31 | 44 | 42 | 40 | 41 | 37 | 28 | 39 | 6 | 22 | 26 |
| 17. | 41 | 43 | 37 | 50 | 37 | 42 | 35 | 29 | 37 | 32 | 38 | 34 |
| 18. | 50 | 43 | 42 | 50 | 46 | 46 | 48 | 50 | 41 | 39 | 31 | 42 |
| 19. | 46 | 29 | 39 | 38 | 33 | 37 | 41 | 30 | 52 | 39 | 29 | 38 |
| 20. | 39 | 30 | 54 | 46 | 36 | 41 | 40 | 17 | 42 | 38 | 12 | 30 |
| 21. | 52 | 34 | 34 | 48 | 40 | 42 | 42 | 31 | 31 | 18 | 31 | 31 |
| 22. | 50 | 42 | 50 | 37 | 48 | 45 | 34 | 31 | 29 | 18 | 41 | 31 |
| 23. | 40 | 39 | 45 | 48 | 47 | 44 | 28 | 47 | 48 | 33 | 41 | 39 |
| 24. | 38 | 44 | 39 | 44 | 46 | 42 | 34 | 32 | 37 | 23 | 48 | 35 |
| 25. | 48 | 44 | 45 | 34 | 33 | 41 | 30 | 24 | 48 | 32 | 48 | 36 |
| 26. | 40 | 45 | 48 | 44 | 33 | 42 | 22 | 30 | 47 | 41 | 40 | 36 |
| 27. | 55 | 40 | 50 | 48 | 51 | 49 | 35 | 43 | 37 | 23 | 35 | 35 |
| Total . . . | 1,121 | 1,064 | 1,158 | 1,024 | 1,009 | 1,075 | 871 | 736 | 899 | 851 | 804 | 834 |
| Average . . | 41.5 | 39.4 | 42.1 | 37.9 | 37.4 | 39.8 | 32.3 | 27.3 | 33.3 | 31.5 | 29.8 | 30.9 |

$$\sigma = 1.882, \sigma M_2 = 0.833.$$

$$\sigma = 1.5, \sigma M_2 = 0.6707.$$

ment, a gain of 22 points; the high group making 10 points; while the low group gains only 5 points. The same ranking results from a comparison of the first 5 trials and the last 5 trials. When the initial trial is compared with the final average score, the median group still makes greatest gain, but the low group is a close second. The initial ability of the high group was greater than that of the other two groups, and we should not be surprised at this negative correlation; while the high group attains a slightly higher final score.

For these groups, the members of which are in no sense homogeneous as to initial ability, past similar activities, or temperamental traits, the first five or ten trials do not give a measure valid for group comparisons. The low and high

groups which have the greater variability do maintain the same relative ranking with right hand; but for the left hand the high group does not catch up with the low group until

TABLE IV
INDIVIDUAL SCORES OF HIGH GROUP

| Practice Period | Right Hand | | | | Left Hand | | | |
|-----------------|-----------------|-----------------|-----------------|---------|-----------------|-----------------|-----------------|---------|
| | <i>a</i> | <i>b</i> | <i>c</i> | Average | <i>a</i> | <i>b</i> | <i>c</i> | Average |
| 1. | 46 | 26 | 31 | 34 | 32 | 21 | 14 | 22 |
| 2. | 50 | 33 | 46 | 43 | 19 | 25 | 23 | 22 |
| 3. | 52 | 38 | 30 | 40 | 42 | 23 | 19 | 28 |
| 4. | 52 | 27 | 20 | 33 | 36 | 39 | 23 | 33 |
| 5. | 60 | 39 | 25 | 42 | 8 | 31 | 8 | 16 |
| 6. | 50 | 24 | 31 | 35 | 26 | 16 | 22 | 21 |
| 7. | 60 | 31 | 46 | 46 | 37 | 22 | 33 | 31 |
| 8. | 48 | 43 | 26 | 39 | 32 | 26 | 28 | 29 |
| 9. | 42 | 34 ₁ | 46 | 41 | 36 | 21 ₁ | 17 | 25 |
| 10. | 52 | 33 | 31 | 39 | 50 | 19 | 24 | 31 |
| 11. | 43 | 39 | 39 | 40 | 50 | 21 | 23 | 27 |
| 12. | 50 | 32 | 34 | 39 | 37 | 30 | 10 | 27 |
| 13. | 56 | 26 ₁ | 32 | 38 | 42 | 21 ₁ | 28 | 32 |
| 14. | 52 | 20 | 25 ₁ | 32 | 41 | 38 | 29 ₁ | 38 |
| 15. | 60 | 40 | 26 | 42 | 46 | 35 | 39 | 30 |
| 16. | 46 | 39 | 38 | 41 | 17 | 29 | 28 | 36 |
| 17. | 62 ₁ | 48 | 19 | 43 | 50 ₁ | 27 | 15 | 24 |
| 18. | 54 | 33 | 24 | 37 | 29 | 19 | 4 | 22 |
| 19. | 60 | 29 | 28 | 39 | 32 | 22 | 30 | 28 |
| 20. | 56 | 46 | 38 | 47 | 33 | 16 | 33 | 29 |
| 21. | 56 | 36 | 31 | 41 | 38 | 19 | 24 | 32 |
| 22. | 56 ₂ | 48 | 38 | 47 | 54 ₂ | 26 | 36 | 34 |
| 23. | 58 | 43 | 31 | 44 | 39 | 19 | 38 | 28 |
| 24. | 56 | 43 | 43 | 47 | 38 | 27 | 32 | 34 |
| 25. | 44 | 35 | 35 | 38 | 44 | 28 | 40 | 35 |
| 26. | 46 | 44 | 27 | 39 | 36 | 33 | 37 | 40 |
| 27. | 54 | 42 | 37 | 44 | 51 | 36 | 27 | 32 |
| Total . . . | 1,421 | 971 | 877 | 1,090 | 995 | 689 | 684 | 786 |
| Average . . | 52.63 | 35.96 | 32.48 | 40.37 | 36.85 | 25.53 | 25.33 | 29.11 |

$$\sigma = 5.388, \sigma M_3 = 3.11.$$

$$\sigma = 5.385, \sigma M_3 = 3.108.$$

after the first ten trials and only succeeds in equalling it in final average score. *C*, however, was most erratic in left hand performance and weighted the group heavily.

While it seems clear that a higher intelligence level makes for superiority in the target test, it is also evident that those who are ranked low in intelligence by a series of tests and scales and by judgmental ratings make great improvement. The graphs suggest that greater and different stimuli are necessary for such a group.

TABLE V
GROUP COMPARISONS

| | 1st Trial | 1st 5 Trials | Last 5 Trials | Final Trial | Average Score | Maximum Score |
|-------------|-----------|--------------|---------------|-------------|---------------|---------------|
| Right Hand: | | | | | | |
| Low..... | 27 | 34.8 | 37 | 32 | 38.9 | 47 |
| Median..... | 27 | 33 | 43.6 | 49 | 39.8 | 49 |
| High..... | 34.3 | 38.4 | 42.5 | 44.3 | 40.4 | 47.3 |
| Left Hand: | | | | | | |
| Low..... | 25 | 24.4 | 32.2 | 32 | 29.5 | 39 |
| Median..... | 21 | 20.8 | 36.2 | 35 | 30.9 | 42 |
| High..... | 22 | 24.2 | 33.8 | 32 | 29.5 | 38 |

INDIVIDUAL LEARNING CURVES

The individual differences and variations in scores, together with the observation of individual attitude and

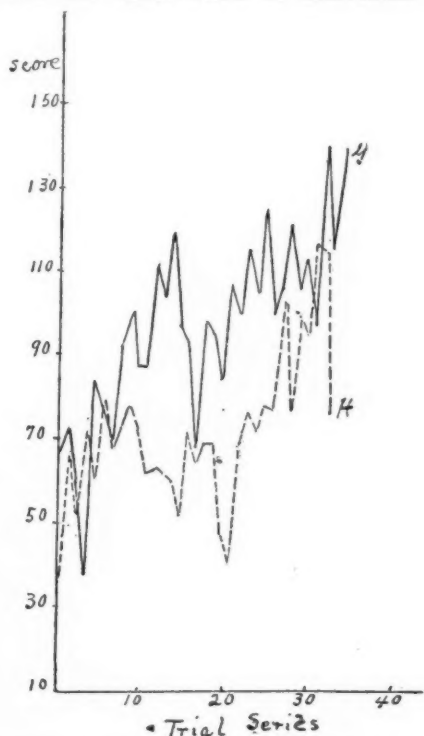


FIG. 4. Learning Curves of Two Stenographers based on daily average scores of 25 throws with left hand.

technique, suggest the possibility of such a test as valuable for study of temperamental types. To follow this more closely and to test the test as one of learning ability, in the fall of 1917 two subjects were given thirty-four practice periods of twenty-five throws with each hand. This gives

TABLE VI

| | G | | H | |
|----------------|-------------------|------------------|------------------|------------------|
| | Right Hand | Left Hand | Right Hand | Left Hand |
| 1. | 45 ₁ | 67 ₁ | 68 ₂ | 37 ₂ |
| 2. | * 32 ₁ | 73 ₁ | 62 | 67 |
| 3. | 61 ₁ | 56 ₁ | 100 ₁ | 52 ₁ |
| 4. | 111 | 37 | 41 | 72 |
| 5. | 82 | 84 | 61 | 61 |
| 6. | 111 | 78 | 52 | 67 |
| 7. | 107 ₁ | 70 ₁ | 58 ₁ | 79 ₁ |
| 8. | 118 | 93 | 78 | 68 |
| 9. | 113 | 101 | 61 | 73 |
| 10. | 107 | 88 | 73 | 78 |
| 11. | 146 | 88 | 77 ₂ | 72 ₂ |
| 12. | 118 | 112 | 70 | 62 |
| 13. | 105 ₁ | 105 ₁ | 58 | 63 |
| 14. | 133 | 119 | 64 | 62 |
| 15. | 127 | 98 | 41 | 60 |
| 16. | 137 | 94 | 57 ₂ | 52 ₂ |
| 17. | 135 | 69 | 42 | 77 |
| 18. | 127 | 99 | 30 ₄ | 65 ₄ |
| 19. | 119 ₁ | 96 ₁ | 49 ₁ | 69 ₁ |
| 20. | 137 | 85 | 55 ₂ | 69 ₂ |
| 21. | 129 ₁ | 108 ₁ | 46 | 48 |
| 22. | 137 | 101 | 80 | 41 |
| 23. | 128 | 116 | 39 | 69 |
| 24. | 127 | 106 | 38 | 77 |
| 25. | 123 | 126 | 56 ₂ | 73 ₂ |
| 26. | 127 | 101 | 61 | 79 |
| 27. | 125 ₁ | 107 ₁ | 54 | 78 |
| 28. | 125 | 122 | 88 ₁ | 104 ₁ |
| 29. | 135 | 107 | 61 ₁ | 78 ₁ |
| 30. | 110 | 114 | 69 | 110 |
| 31. | 107 | 99 | 98 | 106 |
| 32. | 135 ₁ | 141 ₁ | 101 | 118 |
| 33. | 129 | 117 | 107 | 116 |
| 34. | 117 | 141 | 91 | 77 |
| Total..... | 3,925 | 3,318 | 2,186 | 2,479 |
| Average..... | 115.4 | 97.6 | 64.3 | 72.9 |
| σ | 22.8 | 21.46 | 19.14 | 18.92 |

* Arm lame from taking up new physical exercises.

more than twice as much practice as the other groups had. Aside from the increased number of throws and more irregular intervals between practice periods, the method was the same as in other groups.

Both of these subjects were stenographers. *G* was a college graduate, while *H* had only completed high school course, but had more experience in typewriting, and had been more active in outdoor games. Table VI. shows their daily individual scores. The differences are clearly seen in the learning curves in Figs. 4 and 5. *H* seemed quite eager

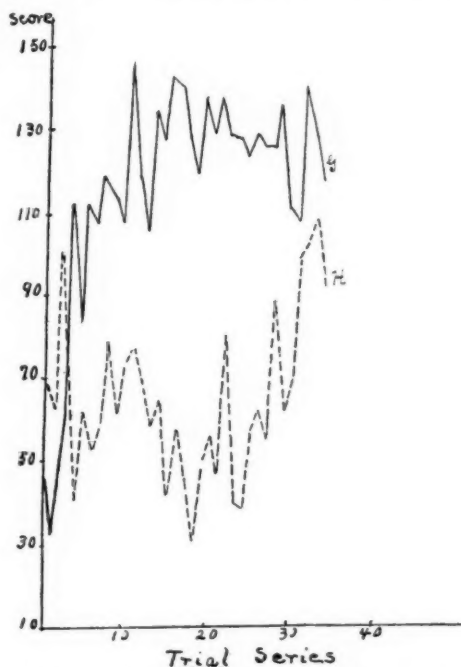


FIG. 5. Learning Curves of Two Stenographers, based on daily average scores of 25 throws with right hand.

to do well, and entered into all such games with a keen, competitive spirit. She knew that *G* was also practicing, but no definite comparisons of their scores were made until the close of the practice work.

H's curve for the right hand shows irregular fluctuations, with marked downward slope in the later practice periods, and lower final capacity than *G*, though initial ability was

superior. The left-hand curve is more nearly normal in type. The curve of *G* is a typical learning curve.

Since there is innate capacity of *H* as evidenced by early trials, such a curve seems explicable only through physiological or temperamental factors. An inquiry was made as to physical well-being at different times during the practice work, and the assurance given that she felt quite well. The daily work as stenographer was maintained at the usual standard. While there is a superficial calm of manner, a marked hesitancy and confusion in speech and in general behavior indicate the highly nervous temperament. Sociological data as to heredity and periods of nervous excitability confirm this.

In a comparative study of the ability of adults and children to learn a maze, Perrin¹ says, "Other personal traits seemed to have influenced the different curves. While we are unable to correlate efficiency in the maze with such temperamental traits or with intelligence in any exact manner, a few general tendencies were noted. The maze taxes the patience and the quick thinker frets about the slow order of things. The individual given to analyzing and theorizing is not necessarily proficient in the maze."

METHODS OF ATTACK

Only gross variations in methods of throwing were evident. A slight shift in point of grasping the dart, an almost imperceptible motion given in hurling, the height of arm at moment of letting go, and the force with which the dart was thrown, would certainly cause varying performances. Observations of the methods used and the general attitude of subject, especially the noting of comments made during the practice period and immediately preceding a trial, suggest three general types of behavior. One analyzes the results, sometimes giving a reason for a poor throw, again expressing disappointment and failures to understand the cause. This would usually result in some change in procedure, more often a steadier

¹ "A Comparison of the Factors Involved in the Maze Learning of Human Adults and Children," *Jour. Ex. Psychol.*, 1916, p. 131.

fixating of the bull's-eye and a more accurate motor adjustment. A consequent success was an added stimulus, and this self-competition proved the best form of incentive. Another type, including 3 of the low group, showed a momentary interest in the result. There was an experience of pleasure over a good throw and sometimes one of dismay over a very unsuccessful attempt. Either might result in added force to the next throwing, which was as likely to fall wide of the mark as to hit it, but rarely in a critical appraisal of the usual technique. A desire to make a better record than some other person seemed the best stimulus for this group, but did not hold from one day to another.

A third class seem intensely interested in achieving success, can scarcely wait for a dart to be withdrawn before throwing, but are overexcited for steady throwing. Failures aggravate the emotional reaction. Self-consciousness as to method and comparative score is a strong factor in their behavior. Members of this class are often of high intelligence, eager to try again, and at various times came back later in a day on which a poor score was made, asking for another trial and stating their belief that they could do better.

An analysis of the single scores during an individual's practice period might show interesting results as to effect of good or poor throws and also might give evidence of a warming-up process or of fatigue effects in later trials. That has not been done with the data yet. These subjects and others who have indulged in several practice periods show a tendency to wish to throw one dart immediately after the other. They give evidence of a physical displeasure over the arrested motion necessary in waiting for the withdrawing of dart by experimenter. There are suggestions of rhythmical reactions on the part of the subject. If the experimenter's reaction time was prolonged, as often happened when the dart was firmly imbedded in target or chanced to stick into the wood, the subject would sometimes throw ahead of time—a danger to be avoided by the observer.

On the other hand, when one throw immediately follows another, a dart will at times stick into the head of another,

again be veered farther away from the center by slight contact with another dart. The question also arises in this case of a segregation of darts forming a larger and more appealing target.

A study of these two forms of procedure and of the sequences of individual throws is being made on other subjects. The effect of an emotional disturbance, more particularly a form of stage fright, is also under observation.

SUMMARY

On the basis of group averages, the high group, representative of normal intelligence as compared with society at large, has greatest initial ability and highest final average. The median group ranks second. The low group, or those with marked mental limitations, has the lowest average. With the standard deviation as a criterion of unreliability, these average scores do not vary by significant amounts.

The low and high groups are characterized by wide variability, while the median group is closely grouped about the central tendency. Scarcity of numbers and the selection of individuals varying so widely as to previous conditioning factors and in daily performance, invalidate generalized comparisons based upon the statistical data.

The learning curves for the low and high groups are characterized by marked fluctuations and valleys rather than plateaus. These valleys occur at different stages of practice, indicating a differentiation of incentives for groups of varying levels of intelligence. The median group has the usual form of learning curve.

While the data indicate the effectiveness of superior intelligence in the acquisition of skill in the target test, there is evidence of great capacity to improve in the upper grade mental defective.

Individual learning curves are suggestive of temperamental types. The analysis of methods of attack and variability in performance, differentiates individual traits other than those dependent upon intellectual ability as it is ordinarily regarded.

PLOTTING EQUATIONS OF THREE VARIABLES IN MENTAL MEASUREMENTS

BY HERBERT A. TOOPS

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The possibility of using a series of curves, common and representative series values of a variable, to represent on ordinary plotting paper the variations of a dependant third variable of a mathematical equation, is not well known. The clearness with which these three-fold relationships can be visualized thereby, and the ease and accuracy with which such charts can be constructed for some of the simpler mathematical equations used by a clinical psychologist, would seem to recommend them to his use.

The two indices, commonly used by the clinician as comparative measures of mental ability, are cases in point. The first of these, the C.I.A. of the Yerkes-Bridges Point Scale, has the equation:

$$\text{C.I.A.} = \frac{\text{Score made by subject}}{\text{Norm for age of subject}} \quad (1)$$

And of the second, the I.Q. of the Terman or Stanford Revision of the Binet Scale:

$$\text{I.Q.} = \frac{\text{Mental age of subject}}{\text{Chronological age}} \quad (2)$$

Each of these equations is the result of a simple division of variables of the first degree. It is quite possible to represent all the relationships of each of these equations, each made up of three variables, by means of a simple series of straight line curves, thus eliminating the use of clumsy tables, logarithms, and the like. Ordinarily we are interested in plotting only representative values (in this case, mostly used values) of one of the variables. The I.Q.'s, for instance, are never plotted for values of chronological age above 16, nor below 3.

The principle used is that of proportionality of similar

right triangles. Let us refer to the dotted lines of Fig. 1. If on the ordinate-axis we lay off a series of decimal values, 0.00 to 1.00 inclusive, in any convenient scale, and then draw a line through the origin of the axes and intersecting the horizontal line which goes through the ordinate (1.00) at the point, a , we shall have the line, Oa . This line, Oa , serves to indicate the quotients of a whole continuous series of

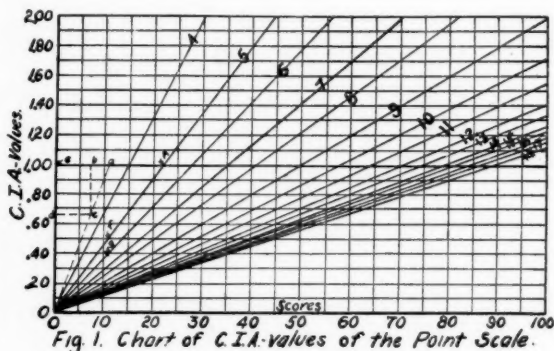


FIG. 1. Chart of C. I. A. values of the Point Scale.

divisions of a variable by a constant quantity. It indicates the quotients obtained by dividing every possible value between e and a successively by the quantity, ea . If we consider any given abscissa value, be , and from the point where the ordinate line of b cuts the line, Oa , project across to the ordinate-axis we find the value, Od , which is the quotient of be/ae . The proof of this and the applicability to mental coefficients will now be shown.

Let b be any point on the horizontal line through the ordinate, 1.00. Then, in the similar triangles, aOe and cOd , $Od/Oe = cd/ae$, but since, $Oe = 1.00$, and $cd = be$,
Therefore,

$$Od = be/ae. \quad (3)$$

Or, Od is the quotient of be divided by ae . This holds true for any point, b , irrespective of its position on the abscissa-axis and also irrespective of the scale in which either ordinates or abscissas are plotted. By extension of the line, Oa , the

straight line curve serves to indicate quotients larger than unity for values of b greater than be as divided by the value, ae . Multiplying the quotients thus obtained by any constant quantity—say by 100 as is commonly done for these mental coefficients—has no effect on the principle involved. We may label the quotients whatever we wish so long as the series of labels is obtained from the quotients of the diagram by multiplication by a constant quantity.

Applying this to our C.I.A. values: If we let the abscissas represent all possible successive scores on the Point Scale from 0 to 100, and draw our line, OA , through the point whose abscissa is the norm for the age 5 and ordinate is 1.00, then the line, OA , will serve to indicate all possible C.I.A. values that might be made by a child five years old for all the scores possible for him to make, within the limits of the chart. Accordingly we label this line a 5-year line. The norm for this age is 22, and a 5-year-old who makes 11 points on this Scale (we see by tracing across the point, f , where the 11-ordinate cuts the line, OA), has a C.I.A. of .50. A 6-year-old who makes the same score, 11, (we see by tracing across from g) has a C.I.A. of .39. In similar manner, each age line is drawn through the score (with C.I.A. of 1.00) which is the norm for that age.¹

The advantages of such a chart are many. It eliminates all computation and secures a distinct saving of time where quotients are needed quickly or where one has many to compute. The scale of the chart may be made large enough to secure any desired degree of accuracy. It can be constructed with the aid merely of pencil and straight edge. If the norms should be changed, the addition of an age line at the proper new norm would allow for the change, a thing not possible with a table of computed values. The interrelations of the coefficients are easily grasped. Ever recurring questions, such as, "What score would it have been necessary for the subject to make in order to have secured such-and-such-a coefficient" are answered by a glance at the

¹ Yerkes, R. M., and Wood, L., 'Methods of Expressing Results of Measurements of Intelligence,' *Jour. of Educ. Psych.*, Vol. VII., No. 10, 1916, pp. 593-606.

chart. We see also why there is so much more variation of C.I.A. values in the lower ages than in the higher ages; for, one point difference in score makes but a difference of little more than one point difference (0.01) in C.I.A. at the age 16, while at the age 4 it makes a difference in C.I.A. of almost 7 points (0.07). This is a great fault of the C.I.A. values. The almost horizontal angle of the higher age curves and the almost vertical angle of the lower age curves make for opposite extremes in their effect upon the C.I.A. values.

A chart constructed in like manner for I.Q. values has already been published.¹ Similar charts used for expressing the quotient of one variable of the first degree divided by another variable also of the first degree, which quotient may then be multiplied by any constant quantity, may as easily be constructed for all such equations.

A further extension of this principle may be made to other simple mathematical equations often used in the psychological clinic. Two examples will suffice to illustrate the application.

Let us consider the Spearman formula for rank correlation,

$$\rho = 1 - \frac{6\Sigma D^2}{n(n^2 - 1)}. \quad (4)$$

This may also be written,

$$\rho = 1 - \frac{6}{n^3 - n} \cdot \Sigma D^2. \quad (5)$$

Now, ΣD^2 has a maximum value when there is a perfect inverse correlation (when $\rho = -1$), or when

$$1 - \left(\frac{6}{n^3 - n} \right) \cdot \Sigma D^2 = -1.$$

That is (solving for ΣD^2), when

$$\Sigma D^2 = \frac{n^3 - n}{3}. \quad (6)$$

The expression (6) is then an expression of the greatest possible deviation for all values of n .

¹ Toops, H. A., and Pintner, R., "A Chart for the Determination of I. Q. Values," *Journal of Delinquency*, Vol. III, No. 6, 1918, p. 272 + chart.

For any given single value of n , the expression, $[6/(n^3 - n)]$ is a constant quantity and with ΣD^2 values calculated, equation (5) is a straight line function which may be plotted on the coördinates—ordinates (ρ) and abscissas (ΣD^2). There will, of course, be a straight line curve for each value of n . In

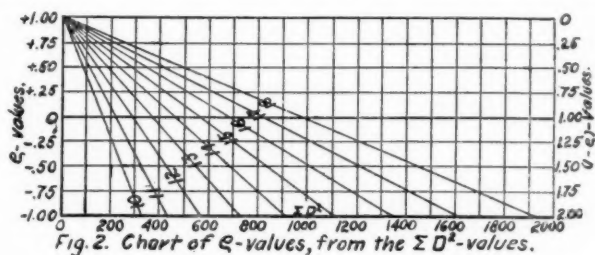


Fig. 2. Chart of ρ -values, from the ΣD^2 -values.

Fig. 2 we have plotted several of these values. Any other curve for any particular value of n desired may be drawn with a ruler and pencil, taking care only that the line pass through the nodal point ($\rho = +1.00$), and through the maximum ΣD^2 on the line ($\rho = -1$), through the $[(n^3 - n)/3]$

TABLE I.
MAXIMUM ΣD^2 -VALUES CORRESPONDING TO VALUES OF n .

| n | Max. ΣD^2 | n | Max. ΣD^2 | n | Max. ΣD^2 | n | Max. ΣD^2 |
|-----|-------------------|-----|-------------------|-----|-------------------|-----|-------------------|
| 9 | 240 | 23 | 4,048 | 37 | 16,872 | 51 | 44,200 |
| 10 | 330 | 24 | 4,600 | 38 | 18,278 | 52 | 46,852 |
| 11 | 440 | 25 | 5,200 | 39 | 19,760 | 53 | 49,608 |
| 12 | 572 | 26 | 5,850 | 40 | 21,320 | 54 | 52,470 |
| 13 | 728 | 27 | 6,552 | 41 | 22,960 | 55 | 55,440 |
| 14 | 910 | 28 | 7,308 | 42 | 24,682 | 56 | 58,520 |
| 15 | 1,120 | 29 | 8,120 | 43 | 26,488 | 57 | 61,712 |
| 16 | 1,360 | 30 | 8,990 | 44 | 28,380 | 58 | 65,052 |
| 17 | 1,632 | 31 | 9,920 | 45 | 30,360 | 59 | 68,440 |
| 18 | 1,938 | 32 | 10,912 | 46 | 32,430 | 60 | 71,980 |
| 19 | 2,280 | 33 | 11,968 | 47 | 34,592 | 61 | 75,640 |
| 20 | 2,660 | 34 | 13,090 | 48 | 36,848 | 62 | 79,422 |
| 21 | 3,080 | 35 | 14,280 | 49 | 39,200 | 63 | 83,328 |
| 22 | 3,542 | 36 | 15,540 | 50 | 41,650 | 64 | 87,360 |

value for the n considered. The maximum ΣD^2 values for plotting the curves with other values of n are given in Table I.

We often desire to know just what is the effect upon the correlation of an apparently aberrant case. This may be

easily found, within certain definite limits to which it may be confined, from Fig. 2, as may be seen from the following considerations: Equation (5) may also be written,

$$(1 - \rho) = \left(\frac{6}{n^3 - n} \right) \cdot (\Sigma D^2). \quad (7)$$

These $(1 - \rho)$ values may be used as ordinates with exactly the same set of curves as for equation (5). Any particular deviation in rank, h , will give to its own particular deviation of the formula a value, $D^2 = h^2$. It will, however, affect the other rankings also, as necessarily some of these are displaced by the aberrant ranking. The maximum effect of this displacement is when the lowest rank of ranking A becomes the highest rank of ranking B , or *vice versa*. Ordinarily, however, only part of the ranks are displaced by the aberrant rank; if the aberrant rank is shifted *down*, all those displaced measures *above* the aberrant measure are each shifted one point in rank, and *vice versa*. The maximum effect, when one extreme measurement of one ranking becomes the other extreme of the other, is thus represented by the expression,

$$D^2 = h^2 + (n - 1) \cdot (1) = (h^2 + n - 1). \quad (8)$$

On a purely chance proposition, the average value of $(n - 1)$ of the above equation would be $(n - 1)/2$; for practical purposes with n very large, the effect of $(n - 1)$ may be disregarded.

We can best grasp the meaning of equation (8) by assuming that the correlation of any two series of measurements would be perfect if it were not for the deviations. Each deviation serves to reduce the correlation by an amount corresponding to a $(1 - \rho)$ value for its representation in ΣD^2 , which must be *at least* (h^2) and *not more than* $(h^2 + n - 1)$. The ρ -limits corresponding to these may be found on Fig. 2 by taking the $(1 - \rho)$ value (righthand ordinates) corresponding to (h^2) and to $(h^2 + n - 1)$ taken as ΣD^2 values (abscissas). Aside from its theoretical interest, such a chart is especially valuable when one has a number of trial correlations to be calculated by this method.

Another more specialized application of the principle is shown by the equation of the P.E. of the above ρ -values.

$$\text{P.E.}\rho = (0.706) \cdot \frac{(1 - \rho^2)}{\sqrt{n}}. \quad (9)$$

Assuming that to be valid or acceptable, a correlation, ρ , must be a times the P.E. ρ , we may construct a series of line curves to show the number of cases necessary with a given ρ to secure a greater reliability, or ratio of $\rho/\text{P.E.}\rho$.

Our assumed measure of reliability of ρ is:

$$a = \frac{\rho}{\text{P.E.}\rho}. \quad (10)$$

Then, in order to be valid, the P.E. ρ must be no more than the value, ρ/a , or

$$\text{P.E.}\rho = \frac{\rho}{a} = (0.706) \cdot \left(\frac{1 - \rho^2}{\sqrt{n}} \right). \quad (11)$$

$$\text{Solving for } n, \rho \sqrt{n} = (0.706) \cdot (a) \cdot (1 - \rho^2)$$

$$n = (0.498) \cdot (a)^2 \cdot [(1 - \rho^2)/\rho]^2. \quad (12)$$

For any particular value of a , the expression $(0.498a^2)$ is a constant quantity. Also the expression $[(1 - \rho^2)/\rho]^2$ may be plotted as abscissas of a simple linear series of values and the appropriate ρ -values to which they refer (not an arithmetical series when plotted) so labeled. The straight line curves of Fig. 3 are labeled from the a -value from which they are derived, and not for the actual values used in plotting them. They can easily be plotted from the value of n when $[(1 - \rho^2)/\rho]^2$ is taken to be 100 in case of each value of a taken. Then, by our principle already given, with the straight line through the origin of the axes and the point,

$$\left[\left(\frac{1 - \rho^2}{\rho} \right)^2 = 100; n = n_{100} \right],$$

the value of $n_{100/A}$ must be $1/A \cdot n_{100}$. Or, taking a concrete example, $n_{50} = 1/2 \cdot n_{100}$. The expression $[(1 - \rho^2)/\rho]^2$ is thus treated always as the independent variable, and as though

it were a simple variable, x , so far as plotting is concerned. An example will best illustrate the use of Fig. 3.

With a correlation, $\rho = 0.4$, we find by tracing across to the left the point, h , where the $(\rho = 0.4)$ -ordinate cuts the

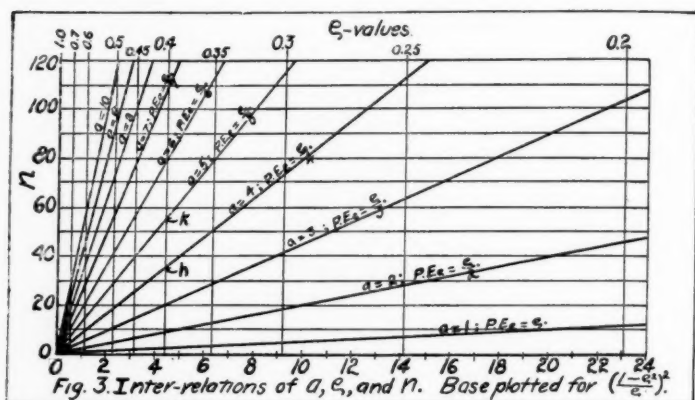


FIG. 3. Interrelations of a , ρ , and n . Base plotted for $[(1 - \rho^2)/\rho]^2$. With a correlation $\rho = 0.4$, we find by tracing across to the left the point, h , that 35 cases are necessary in order that the P. E. ρ shall be only one fourth as large as ρ . In order to reduce the ratio so that the P. E. ρ shall be only $1/5$ of ρ , we project across the point, k , and find that 55 cases will be necessary.

$(a = 4)$ -curve that 35 cases will be needed in order that the P.E. ρ shall be only one fourth as large as ρ . Since the P.E. $\rho = \rho/a$, in this case it is $0.4/4 = 0.1$. Now, if we wish to know how many cases we need with the same correlation (0.4) to secure an a -value of 5, we have but to project the point, k , across to the left, whereupon we find that 55 cases will be necessary. The P.E. ρ here is 0.08.

By this procedure we have reduced an equation of three variables and of the second degree to a chart of straight line curves.

In like manner we may plot any equation of the general formula, $y = A \cdot Z^v \cdot X^w$, in which A is a constant quantity, Z^v is a variable of any degree of which commonly used and representative curve values are desired, X^w is either a complicated or simple expression which is to be treated in plotting

as a simple variable, x . When X^w is plotted along the lower base line of our plotting paper, we may then plot along the upper base line of the plotting paper as a geometrical variable a series of values (which may be computed for intermediate values), corresponding to the linear or arithmetical series below, the variations of the variable expression (not necessarily x as shown above) which produces X^w . The individual curves are then to be labeled from the $(Z)^1$ value from which they were obtained. In all cases, the variations should be examined to see whether they may profitably be plotted as straight line curves. In general, it is unprofitable if w is larger than 3, unless only small ranges are taken of the variable expression which produces X^w ; v should also preferably be small or unity.

Fig. 4 illustrates a method of plotting ($y = a \cdot x^2$) in such manner that not only common and representative values of

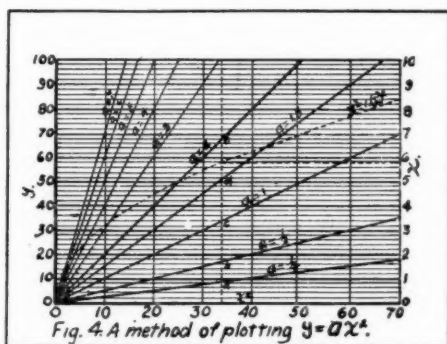


FIG. 4. A method of plotting $y = ax^2$. An x -value of 5.8, on the right-hand scale, traced across to the left until it hits the parabola and then projected downwards to the base gives us an x^2 -value of 33.6. Projected upward until hitting the $(a = 1/4)$ -line at a and then projected to the left we secure a y -value of 18.4. That is, for the equation, $y = 1/4 x^2$; when x is 5.8, then y is 18.4.

x may be used but also every possible value. This is secured by plotting the curve of $x^2 = 1 \cdot (x)^2$ in such manner that the curve represents the square of the x -values which are plotted on the right-hand border, and thus being able to project the x^2 -values to the base line. From the dotted lines

of the figure, we see that with an x -value of 5.8 we secure an x^2 -value of 33.6; this ordinate intersects the ($a = \frac{1}{4}$) straight line curve at the point, a , corresponding to a y -value, 8.4. This means that for the equation, $y = \frac{1}{4} x^2$, when x is 5.8, the value of y is 8.4. From the intersection of this ordinate with the other curves at points, b, c, d, e , we can readily find the value of y when the value of a respectively is, $\frac{1}{2}, 1, 1.5$, and 2 . The accuracy of the plotting of the curve, $x^2 = 1 \cdot (x)^2$ can always be tested by comparing the x^2 -values determined from it with the value of $y_{(a-1)}$ determined by the intersection, c ; the line, $a = 1$, being very carefully plotted.